Image and Video Recovery

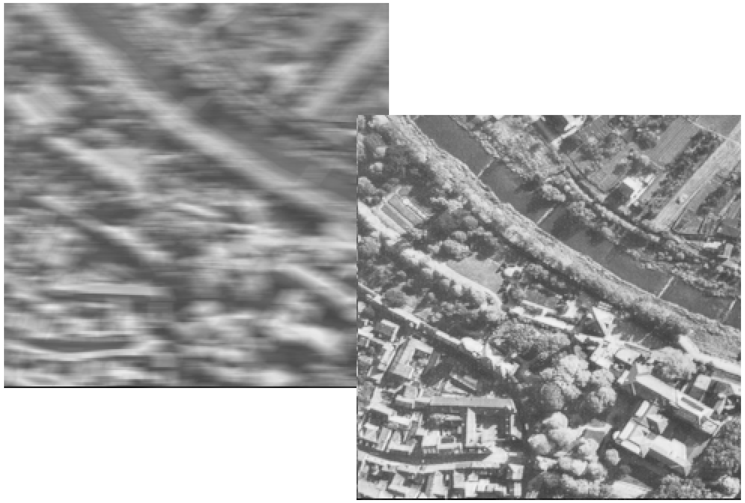

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Image and Video Recovery

Part I:

- Image Restoration
  - deterministic approaches
  - stochastic approaches

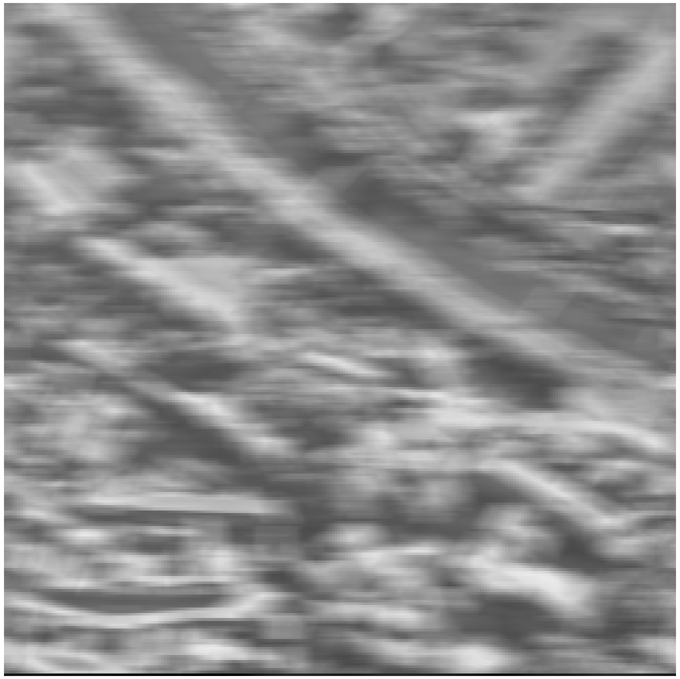
Part II:

- Recovery of Compressed Images and Video
- Concealment of Compressed Images and Video
- Video Resolution Enhancement

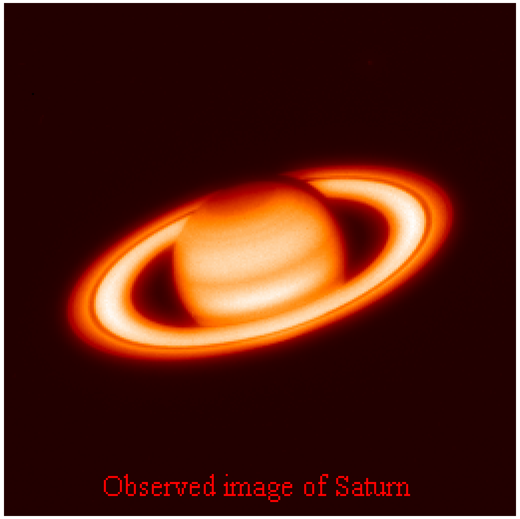
Introduction

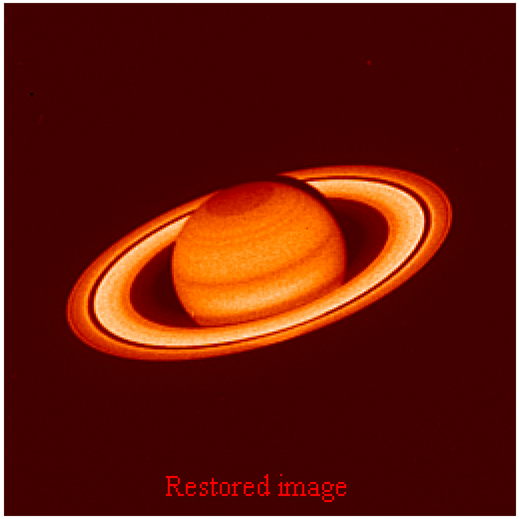
- Recovery vs. Restoration vs. Enhancement
- History of the Field
- Classification of Approaches
- “Classical” Applications
- “New” Applications

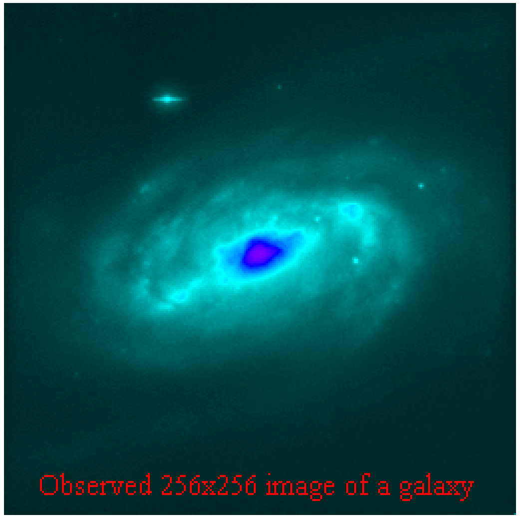
The Image Restoration Problem

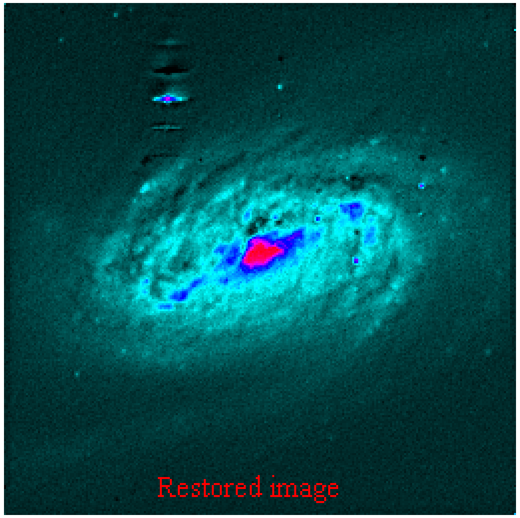


The Image Restoration Problem









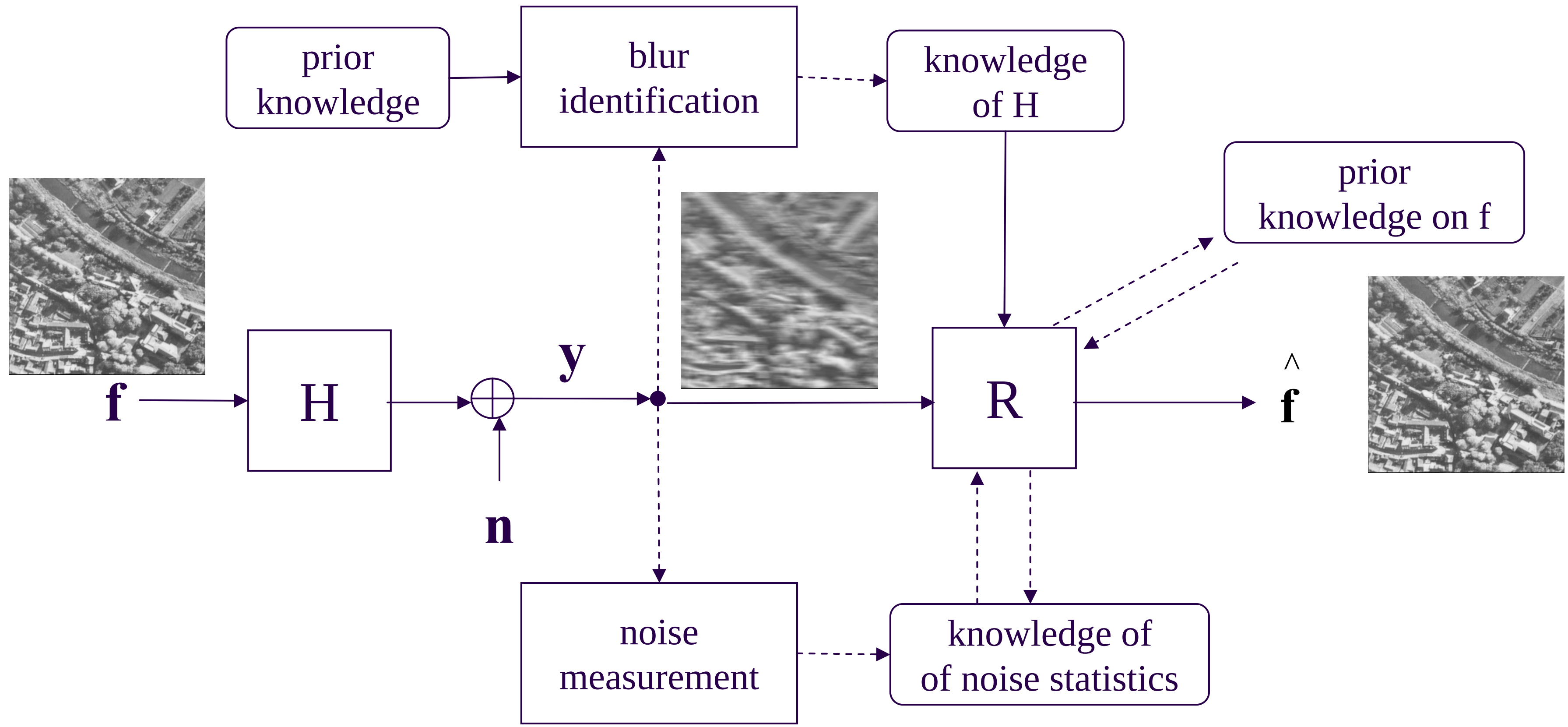
The Image Restoration Problem

original

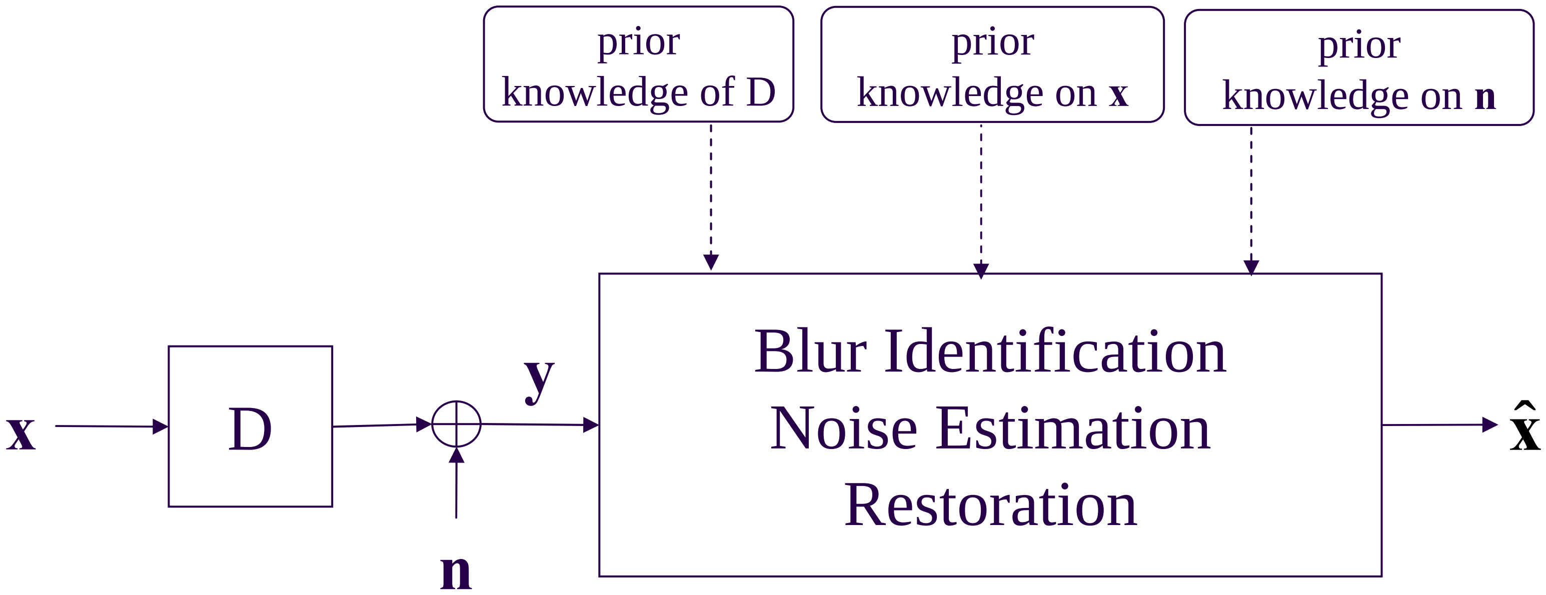
degraded

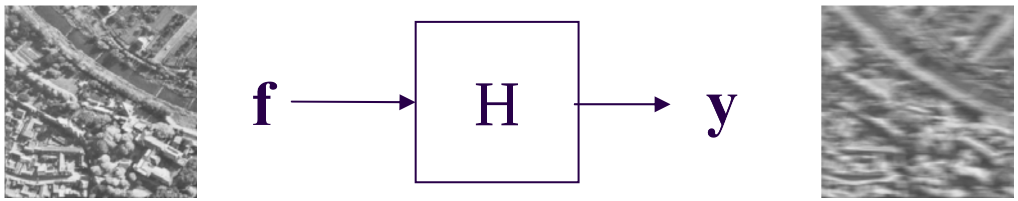
restored

Degradation / Restoration System



Degradation/ Restoration System





Known	Problem Type
H, y	restoration -- an inverse problem
f, y	system identification
f, H	system implementation
y	blind restoration
y, H partially	semi-blind restoration

Motion estimation
Disparity estimation
Boundary detection through differentiation

} *Inverse problems*

Applications

- space exploration, HST
- medicine (diagnostic x-rays, sinograms)
- nondestructive testing
- commercial, digital photography
- (video) printing
- resolution enhancement
- multi-channel/spectral recovery
- error concealment
- restoration of compressed images

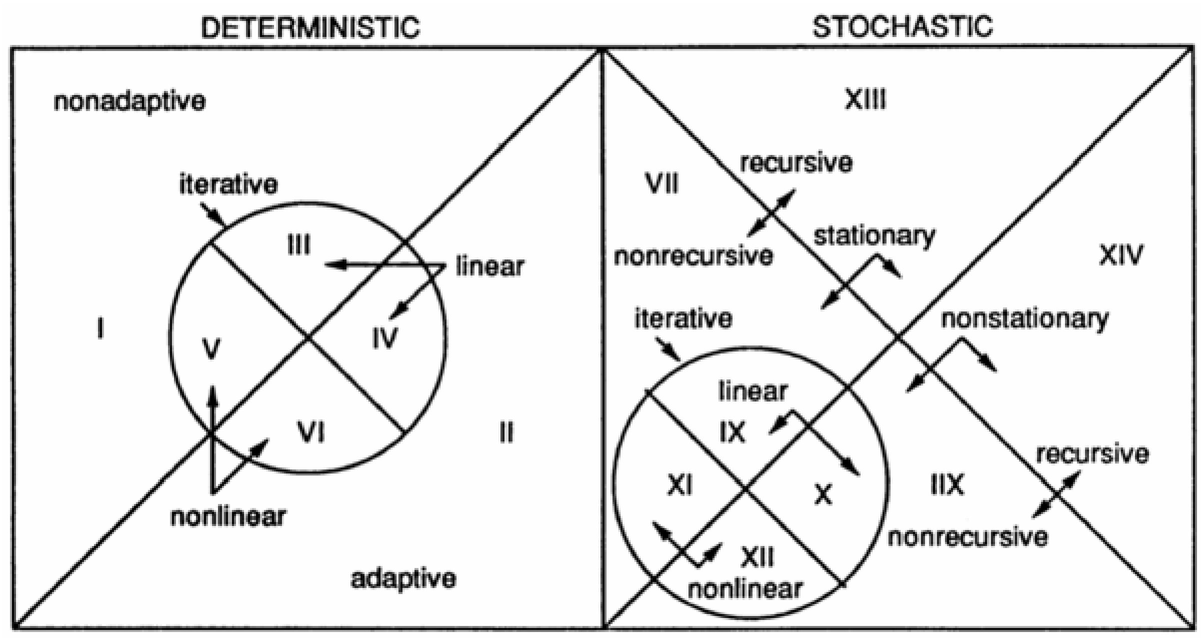
Types of Degradation

- motion
- atmospheric turbulence
- out-of-focus lens
- finite resolution of instruments
- quantization
- transmission errors
- noise

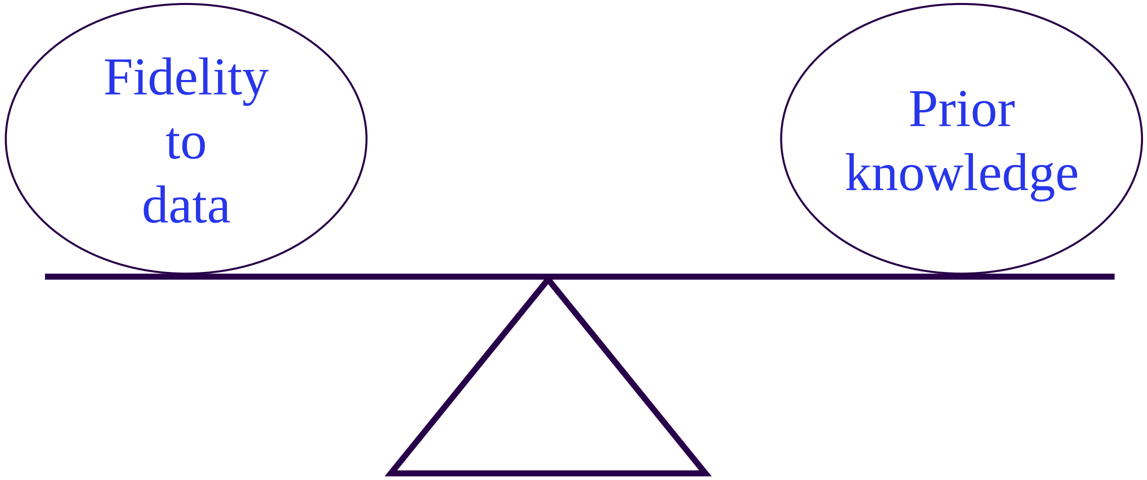
Steps in Restoration

- Choose appropriate degradation model
  - (non)-linear, space (in)variant
  - noise additive, signal (in)dependent
- Regularize the problem
- Choose appropriate solution approach
  - direct, iterative, recursive, spatial domain, frequency domain

Classification of Restoration Techniques



Regularization Principle



Degradation Model

$$y(i, j) = H\left[f(i, j)\right] + n(i, j)$$

noisy-blurred observed image — $y(i, j)$

degradation operator — H

source or original image — $f(i, j)$

noise component — $n(i, j)$

In many applications H[ ] can be well approximated by an Linear Space-Invariant (LSI) system and the noise by an additive and signal independent process

Representative Degradations

- 1-D Motion: $$h(i)=\begin{cases}\dfrac{1}{L} & \text{for } -\dfrac{L}{2}\le i\le\dfrac{L}{2}\\ 0 & \text{otherwise}\end{cases}$$
- Atmospheric turbulence: $$h(i,j)=K\exp\left(-\frac{i^2+j^2}{2\sigma^2}\right)$$
- Out of focus: $$h(i,j)=\begin{cases}\dfrac{1}{\pi R} & \text{for } \sqrt{i^2+j^2}\le R\\ 0 & \text{otherwise}\end{cases}$$
- Pill-box: $$h(i,j)=\begin{cases}\dfrac{1}{L^2} & \text{for } -\dfrac{L}{2}\le i,j\le\dfrac{L}{2}\\ 0 & \text{otherwise}\end{cases}$$

Objective degradation/restoration metrics

- Blurred Signal-to-Noise Ratio (BSNR)

$$\mathrm{BSNR} = 10\log_{10}\left\{ \frac{\frac{1}{MN}\sum_i \sum_j \left[g(i,j) - \bar{g}(i,j) \right]^2}{\sigma_n^2} \right\}$$

$$g(i,j) = y(i,j) - n(i,j)$$

σ_n^2 : noise variance

$$\bar{g}(i,j) = E\{g(i,j)\}$$

- Improvement in Signal-to-Noise Ratio (ISNR)

$$\mathrm{ISNR} = 10\log_{10}\left\{ \frac{\sum_i \sum_j \left[f(i,j) - y(i,j) \right]^2}{\sum_i \sum_j \left[f(i,j) - \hat{f}(i,j) \right]^2} \right\}$$

Matrix-vector notation

- By stacking (lexicographically) the observed image into a vector

$$\mathbf{y} = \mathbf{H}\mathbf{f} + \mathbf{n}$$

- LSI degradation model

$$y(i,j) = \sum_{m=0}^{M-1}\sum_{n=0}^{M-1} f(m,n)h(i-m, j-n) + n(i,j) = f(i,j) * h(i,j) + n(i,j)$$

  - $\mathbf{H}$ is a ***block circulant*** matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_{M-1} & \cdots & \mathbf{H}_1 \\ \mathbf{H}_1 & \mathbf{H}_0 & \cdots & \mathbf{H}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{M-1} & \mathbf{H}_{M-2} & \cdots & \mathbf{H}_0 \end{bmatrix}$$

Spectral Properties of Block Circulant Matrices

$$\mathbf{H} = \mathbf{W}\mathbf{D}\mathbf{W}^{-1} \Rightarrow \mathbf{D} = \mathbf{W}^{-1}\mathbf{H}\mathbf{W}$$

diagonal with elements the stacked values of the 2D DFT of $h(i,j)$

stacked 2D DFT of the image $f(i,j)$

$$\mathbf{y} = \mathbf{H}\mathbf{f} + \mathbf{n} \Rightarrow \mathbf{y} = \mathbf{W}\mathbf{D}\mathbf{W}^{-1}\mathbf{f} + \mathbf{n} \Rightarrow \mathbf{W}^{-1}\mathbf{y} = \mathbf{D}\mathbf{W}^{-1}\mathbf{f} + \mathbf{W}^{-1}\mathbf{n} \Rightarrow$$

Discrete Frequency Domain Representation

$$Y(u,v) = H(u,v)F(u,v) + N(u,v), \quad u,v = 0,1,\ldots,M-1$$

Inverse Filter

- minimize

$$J(\mathbf{f}) = \|\mathbf{n}(\mathbf{f})\|^2 = \|\mathbf{y} - \mathbf{H}\mathbf{f}\|^2$$

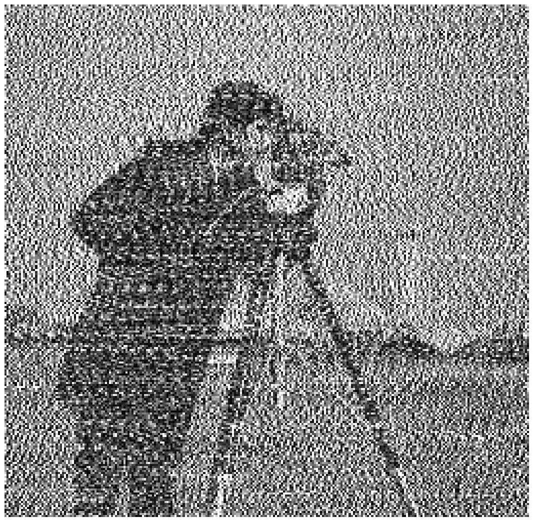
- $$\frac{\partial J(\mathbf{f})}{\partial \mathbf{f}} = 0 \Rightarrow \mathbf{H}^T\mathbf{H}\mathbf{f} = \mathbf{H}^T\mathbf{y} \Rightarrow \mathbf{f} = (\mathbf{H}^T\mathbf{H})^+\mathbf{H}\mathbf{y}$$

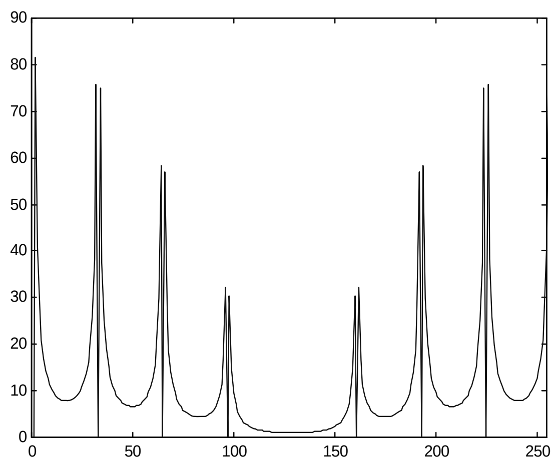
- $\mathbf{H}$ circulant:

$$F(u,v) = \begin{cases} \dfrac{H^*(u,v)Y(u,v)}{|H(u,v)|^2} & |H(u,v)| \neq 0 \ (\geq T) \\ 0 & |H(u,v)| = 0 \ (< T) \end{cases}$$

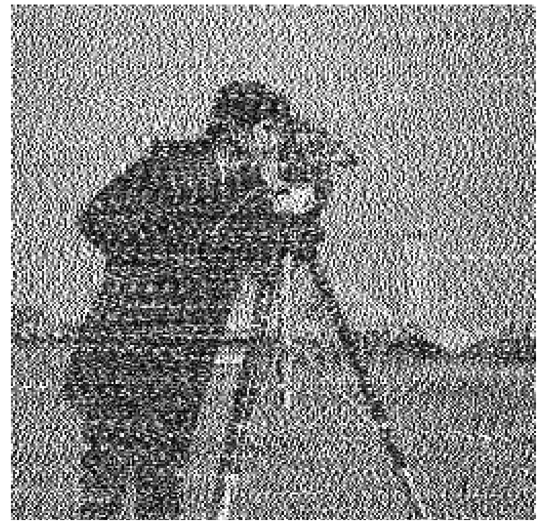
Thresholded inverse filter, 1D blur over 8 pixels, BSNR=20 dB.

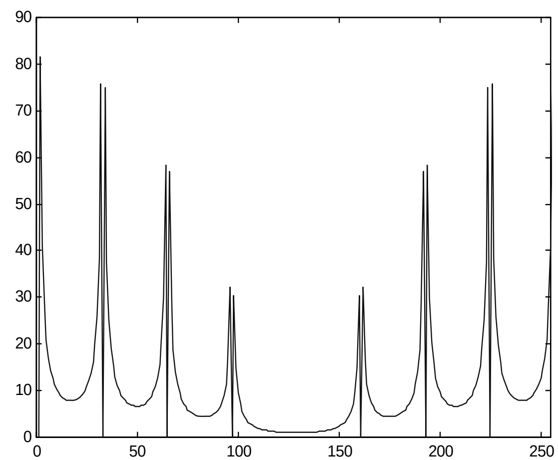
$T=10^{-16}$





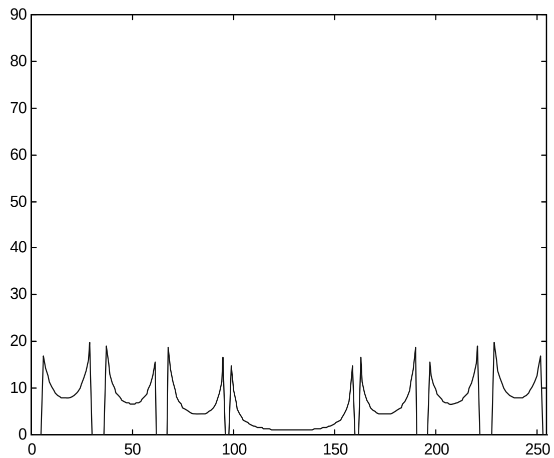
T=.01



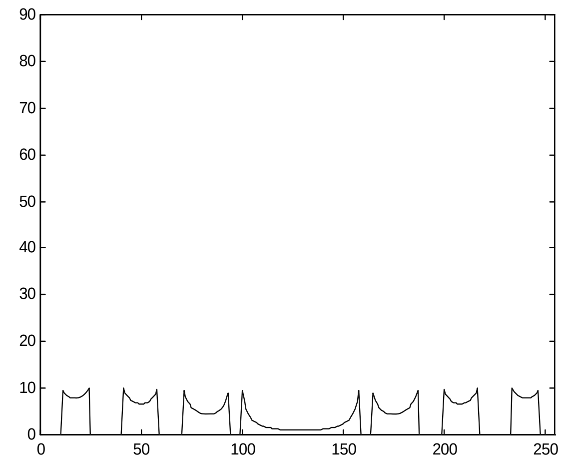


Thresholded inverse filter, 1D blur over 8 pixels, BSNR=20 dB.

T=.05



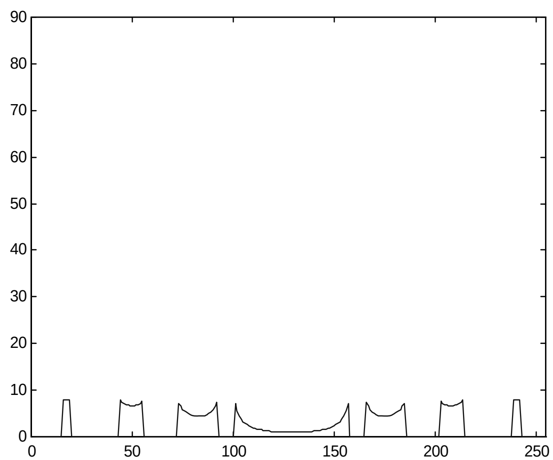
T=.1

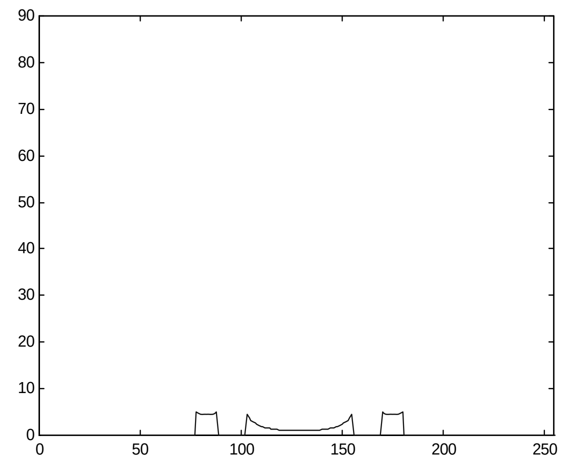


Thresholded inverse filter, 1D blur over 8 pixels, BSNR=20 dB.

T=.125

T=.2





Constrained Least-Squares Filter

- minimize

  $$J(\mathbf{f}) = \|\mathbf{n}(\mathbf{f})\|^2 = \|\mathbf{y} - \mathbf{Hf}\|^2$$

  subject to

  $$\|\mathbf{Cf}\|^2 < \varepsilon$$

- $\min\left(\|\mathbf{y} - \mathbf{Hf}\|^2 + \alpha\|\mathbf{Cf}\|^2\right) \Rightarrow \quad \mathbf{f} = \left(\mathbf{H}^{\mathbf{T}}\mathbf{H} + \alpha\mathbf{C}^{\mathbf{T}}\mathbf{C}\right)^{+}\mathbf{H}^{\mathbf{T}}\mathbf{y}$

- $\mathbf{C}$ is a high-pass filter, such as the 2D Laplacian

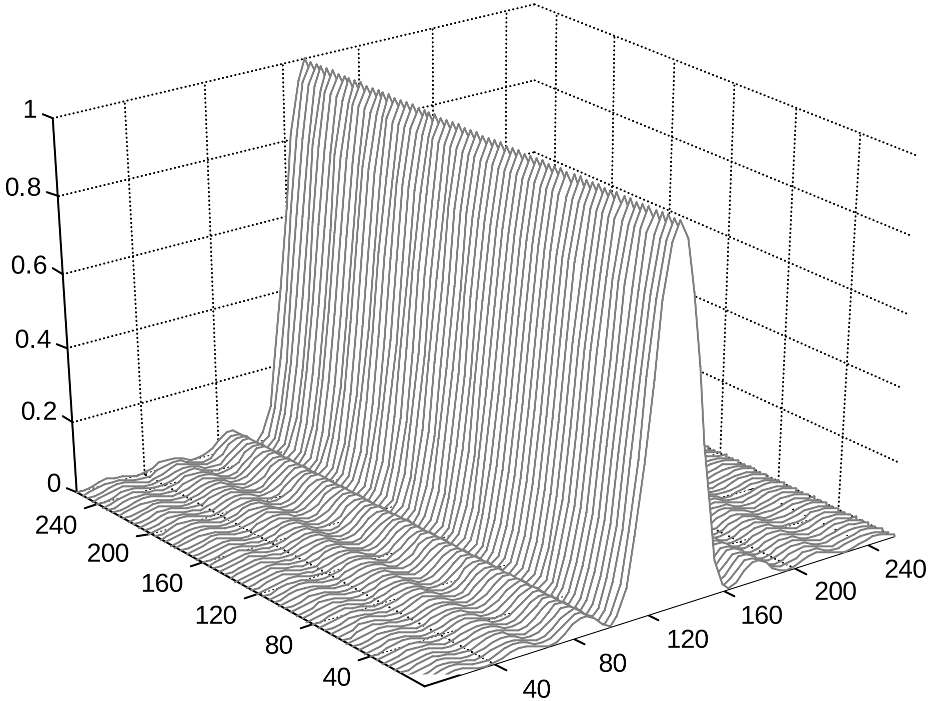
- for $\mathbf{H}$ and $\mathbf{C}$ circulant:

$$F(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \alpha|C(u,v)|^2}Y(u,v)$$

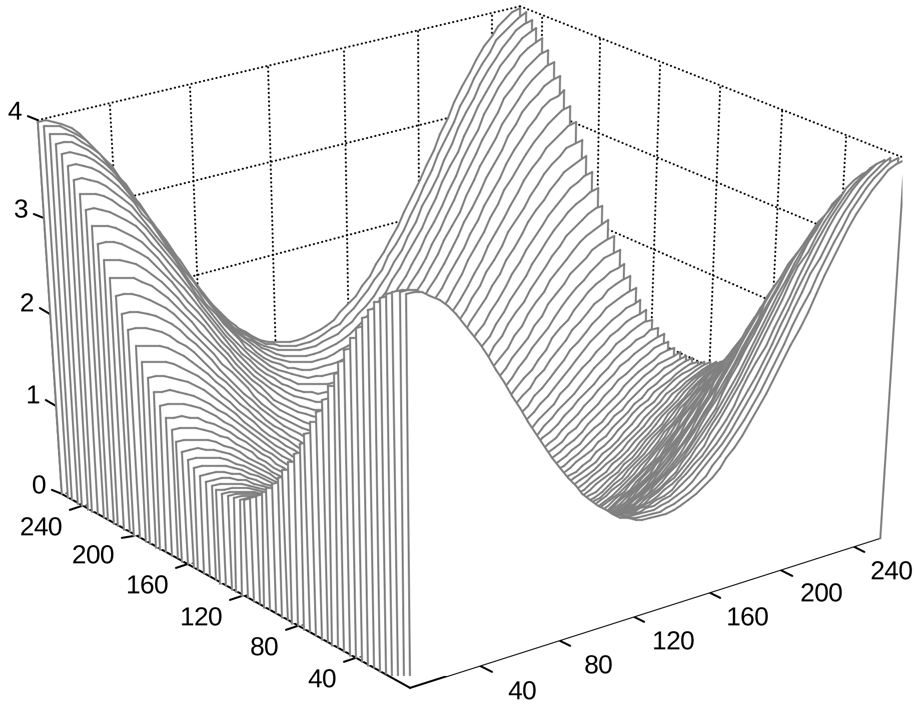
C 2D Laplacian

$$\mathbf{C} = \begin{bmatrix} 0.00 & 0.25 & 0.00 \\ 0.25 & -1.00 & 0.25 \\ 0.00 & 0.25 & 0.00 \end{bmatrix}$$

$|H(u,v)|^2$ of horizontal motion blur over 8 pixels

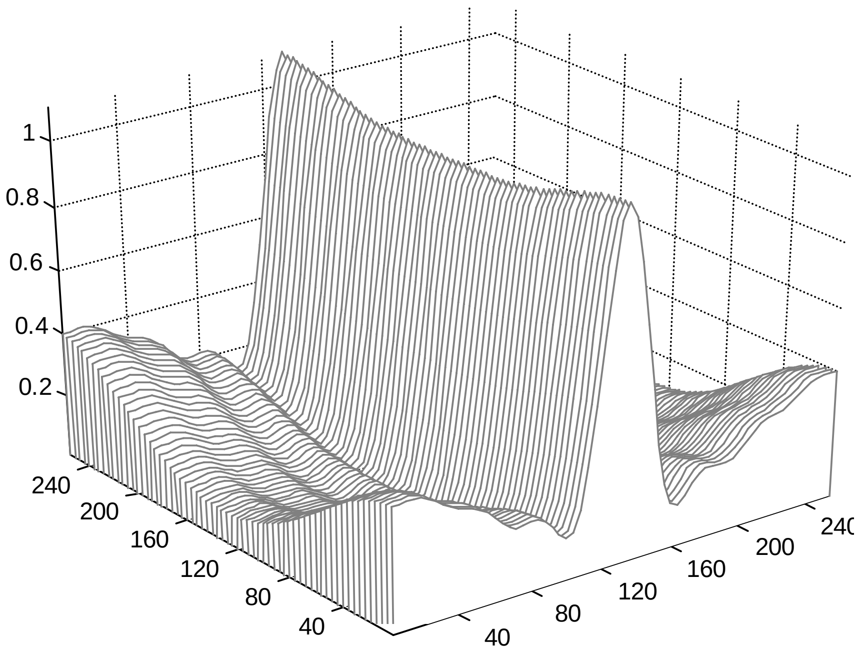


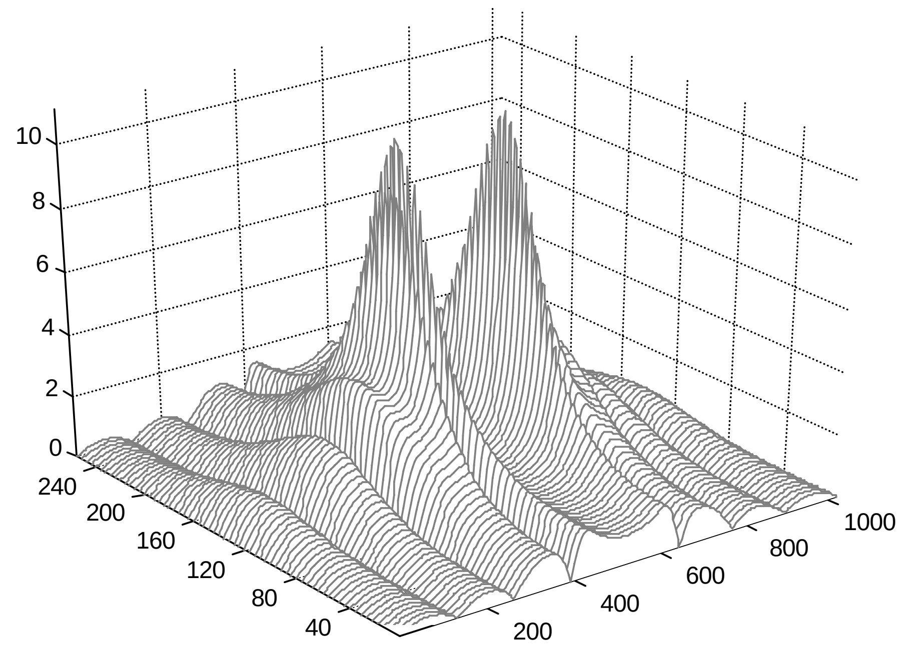
Magnitude squared of the frequency response of a 2D Laplacian $|C(u,v)|^2$



$$\left|H(u,v)\right|^2 + .1\cdot\left|C(u,v)\right|^2$$

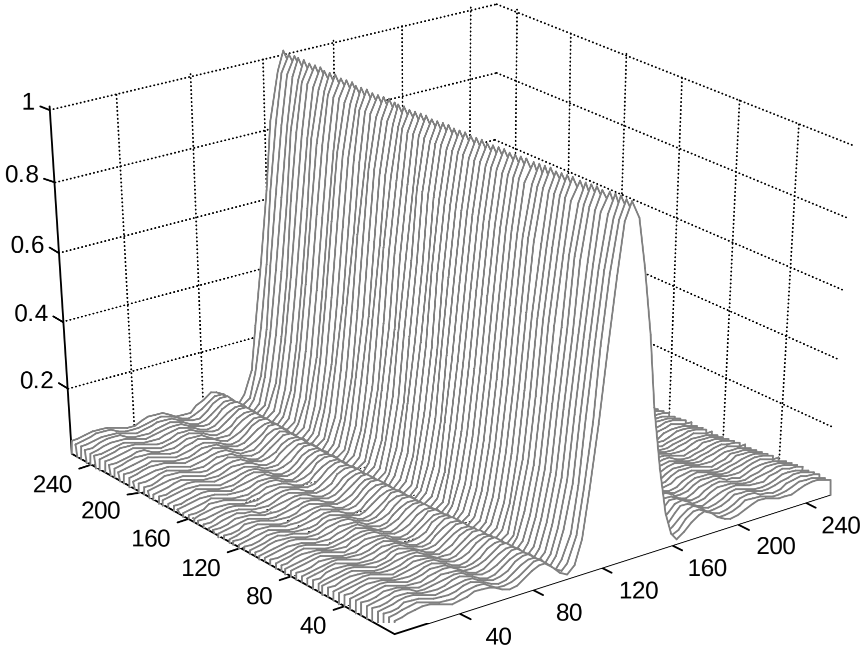
$$\left|\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + .1\cdot\left|C(u,v)\right|^2}\right|$$

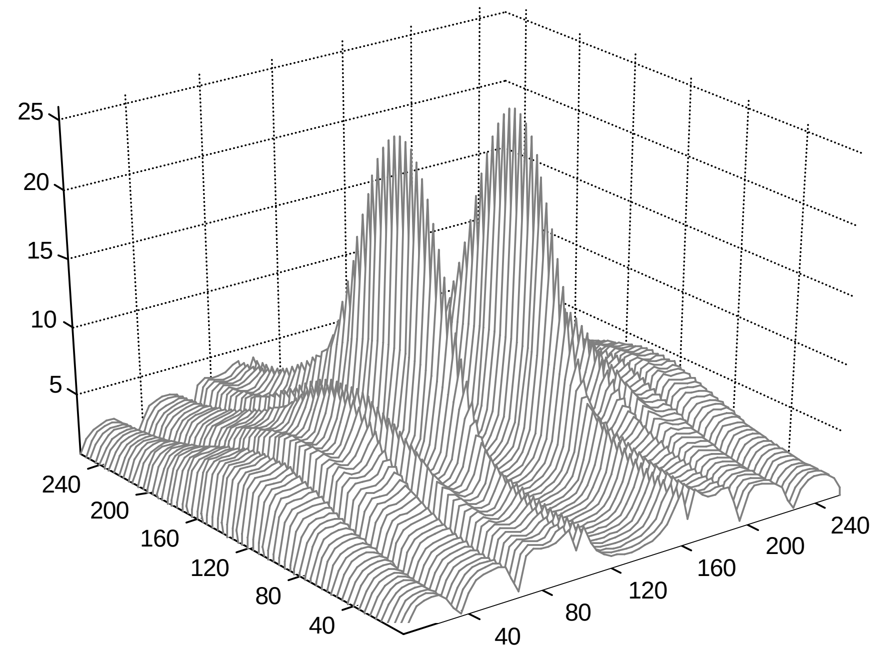




$$\left|H(u,v)\right|^2 + .01 \cdot \left|C(u,v)\right|^2$$

$$\left|\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + .01 \cdot \left|C(u,v)\right|^2}\right|$$





Set Theoretic Approach

- Find a solution belonging to both sets

$$Q_{\mathbf{f}|\mathbf{y}} = \{\mathbf{f} \mid \|\mathbf{y} - \mathbf{H}\mathbf{f}\|^2 \leq E^2\}$$

$$Q_{\mathbf{f}} = \{\mathbf{f} \mid \|\mathbf{C}\mathbf{f}\|^2 \leq \varepsilon^2\}$$

- ***One*** Solution Approach: Center of ellipsoid bounding the intersection

$$\mathbf{f} = \left(\mathbf{H}^T\mathbf{H} + \alpha\mathbf{C}^T\mathbf{C}\right)^{+} \mathbf{H}^T\mathbf{y} \qquad \alpha = (\varepsilon / E)^2$$

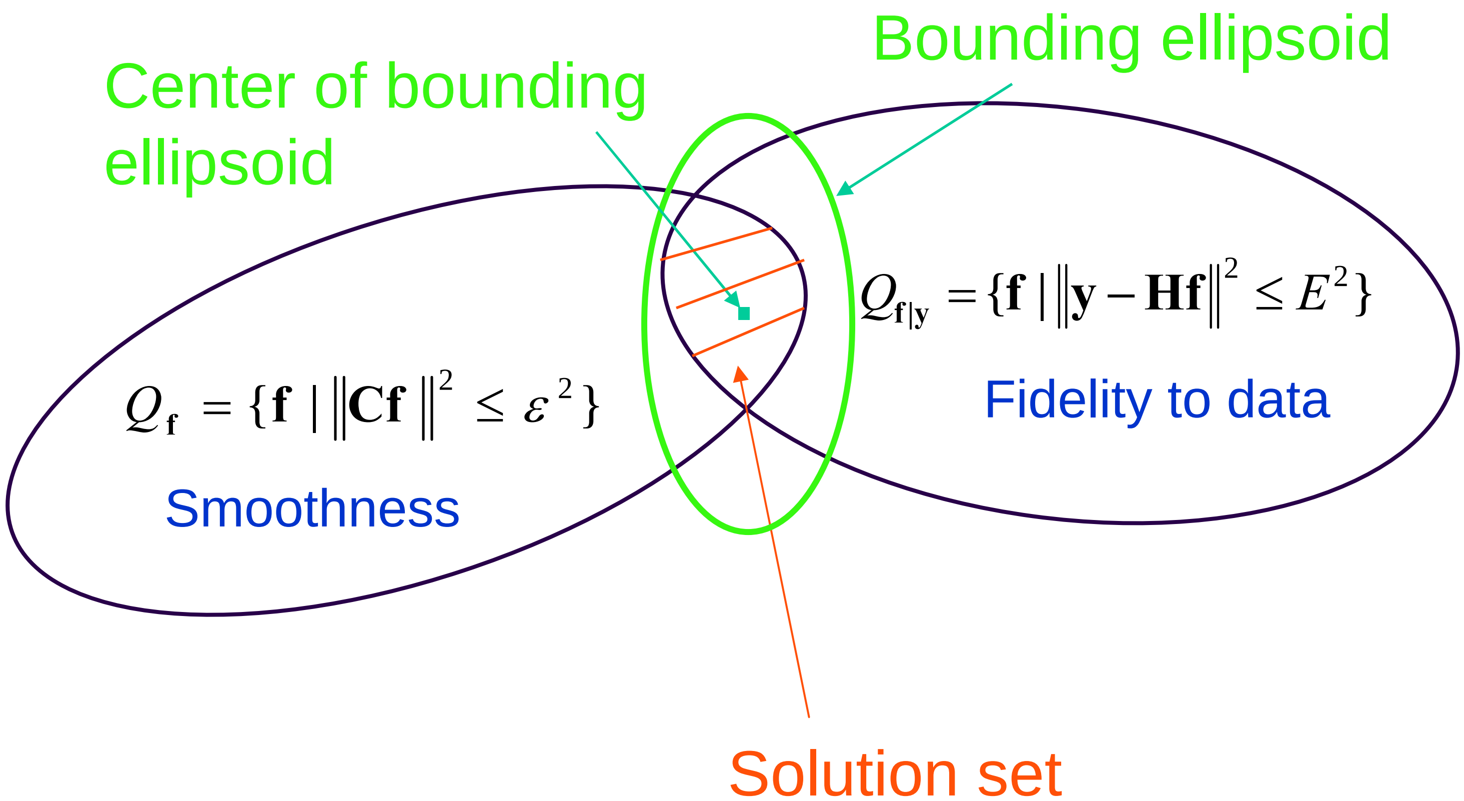
- ***Another*** Solution Approach: Alternate projections onto convex sets

$$\mathbf{f}_{\mathbf{k+1}} = P_1 P_2 \mathbf{f}_{\mathbf{k}}$$

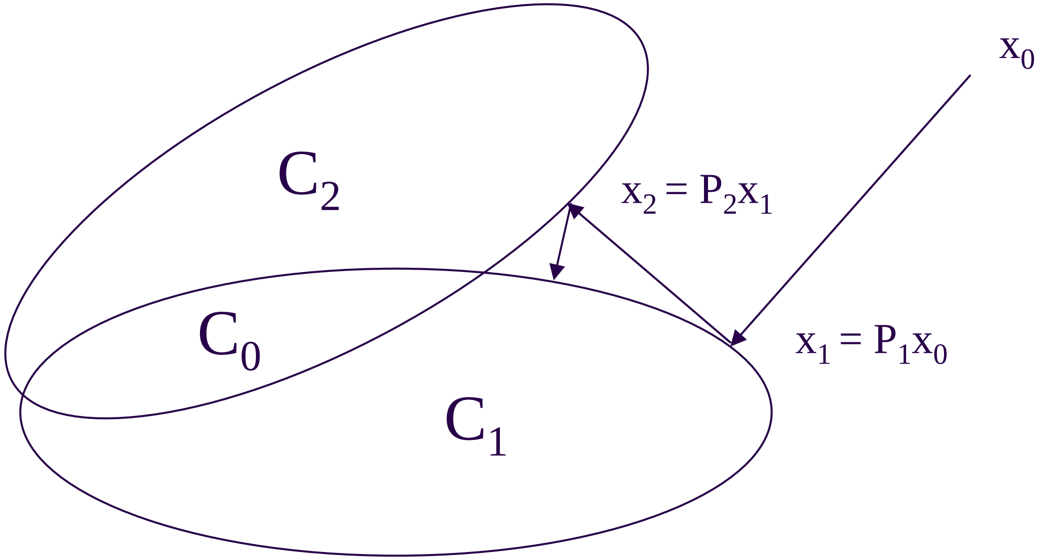
$$P_1\mathbf{f} = \mathbf{f} + \lambda_1\left(\mathbf{I} + \lambda_1\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T(\mathbf{y} - \mathbf{H}\mathbf{f})$$

$$P_2\mathbf{f} = [\mathbf{I} - \lambda_2\left(\mathbf{I} + \lambda_2\mathbf{C}^T\mathbf{C}\right)^{-1}\mathbf{C}^T\mathbf{C}]\mathbf{f}$$

Set Theoretic Recovery Principle



POCS Algorithm



For C_i, $i = 1, 2, \ldots, m$, convex and closed

$$\mathbf{f}_k = P_m \ldots P_2 P_1 \mathbf{f}_{k-1}, \quad \mathbf{f}_0 \text{ arbitrary},$$

then

$$\mathbf{f}_k \to \mathbf{f}^* \in C_0 \equiv \bigcap_{i=1}^{m} C_i.$$

Iterative Restoration Algorithms

- There is no need to explicitly implement the inverse of an operator.
- The restoration process is monitored as it progresses.
- The number of iterations can be used as a means of regularization
- The effects of noise can be controlled in each iteration.
- They can be applied in cases of spatially varying or nonlinear degradations or in cases where the type of degradation is completely unknown (blind restoration).

Basic Approach

Find the root of $\Phi(\mathbf{f})$

Successive Approximations Iteration

$$\mathbf{f}_0 = \mathbf{0}$$

$$\mathbf{f}_{k+1} = \mathbf{f}_k + \beta\,\Phi(\mathbf{f}_k)$$

$$= \Psi(\mathbf{f}_k)$$

Basic Approach

Find root(s) of $\Phi(\mathbf{f})$

Successive Approximations Iteration

$$\begin{aligned} \mathbf{f}_0 &= \mathbf{0} \\ \mathbf{f}_{k+1} &= \mathbf{f}_k + \beta\Phi(\mathbf{f}_k) \\ &= \Psi(\mathbf{f}_k) \end{aligned}$$

Convergence

The successive approximations iteration converges to the unique fixed point $\mathbf{f}^*$, i.e., $\Psi(\mathbf{f}^*) = \mathbf{f}^*$, if $\Psi(\mathbf{f})$ is a ***contraction***

$\left\|\Psi(\mathbf{f}_1) - \Psi(\mathbf{f}_2)\right\| \leq \eta\left\|\mathbf{f}_1 - \mathbf{f}_2\right\|$ for $\eta \leq 1$ and any norm $\|\cdot\|$

Basic Approach

Find root(s) of $\Phi(\mathbf{f})$

Successive Approximations Iteration with Constraints

$$\mathbf{f}_0 = \mathbf{0}$$

$$\tilde{\mathbf{f}}_{\mathbf{k}} = C\,\mathbf{f}_{\mathbf{k}}$$

Constraint or projection operator

$$\mathbf{f}_{\mathbf{k+1}} = \tilde{\mathbf{f}}_{\mathbf{k}} + \beta\Phi(\tilde{\mathbf{f}}_{\mathbf{k}}) = \Psi(C\mathbf{f}_{\mathbf{k}})$$

Convergence

The successive approximations iteration converges to the unique fixed point if the concatenation of operators ΨC is a ***contraction***

Basic Iteration

$$\Phi(\mathbf{f}) = \mathbf{y} - \mathbf{Hf}$$

$$\mathbf{f_{k+1}} = \mathbf{f_k} + \beta(\mathbf{y} - \mathbf{Hf_k}) = \beta\,\mathbf{y} + (\mathbf{I} - \beta\,\mathbf{H})\mathbf{f_k}$$

Frequency Domain Iteration (H block circulant)

$$F_{k+1}(u,v) = \beta\,Y(u,v) + (1 - \beta\,H(u,v))F_k(u,v)$$

Convergence

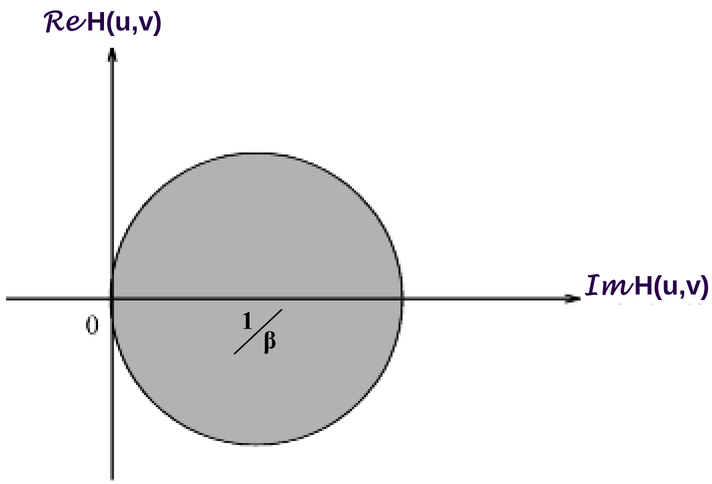
Restoration filter

$$F_k(u,v) = R_k(u,v)Y(u,v)$$

$$R_k(u,v) = \beta \sum_{l=0}^{k-1} \left(1-\beta H(u,v)\right)^l$$

if $\left|1-\beta H(u,v)\right| < 1$

or $0 < \beta < \dfrac{2}{H_{\max}(u,v)}$



$$\lim_{k\to\infty} R_k(u,v) = \lim_{k\to\infty} \beta \frac{1-\left(1-\beta H(u,v)\right)^k}{1-\left(1-\beta H(u,v)\right)} = \begin{cases} \dfrac{1}{H(u,v)} & H(u,v) \neq 0 \\ k\beta & H(u,v) = 0 \end{cases}$$

Least Squares (LS) Iteration

$$\Phi(\mathbf{f}) = \frac{1}{2}\nabla_{\mathbf{f}}\ \|\mathbf{y}-\mathbf{Hf}\|^{2}$$

$$\mathbf{f_{k+1}} = \mathbf{f_k} + \beta\,\mathbf{H}^{\mathbf{T}}(\mathbf{y}-\mathbf{Hf_k})$$

$$= \beta\,\mathbf{H}^{\mathbf{T}}\mathbf{y} + (\mathbf{I}-\beta\,\mathbf{H}^{\mathbf{T}}\mathbf{H})\mathbf{f_k}$$

Frequency Domain Iteration (H block circulant)

$$F_{k+1}(u,v) = \beta H^{*}(u,v)Y(u,v) + \left(1-\beta\left|H(u,v)\right|^{2}\right)F_k(u,v)$$

Convergence

$$R_k(u,v) = \beta \sum_{l=0}^{k-1} \left(1 - \beta |H(u,v)|^2\right)^l H^*(u,v)$$

$$= \beta \frac{1-(1-\beta|H(u,v)|^2)^k}{1-(1-\beta|H(u,v)|^2)} H^*(u,v)$$

sufficient condition for convergence

$$\left|1 - \beta |H(u,v)|^2\right| < 1, \quad \text{or} \quad 0 < \beta < \frac{2}{\max_{u,v} |H(u,v)|^2}$$

$$\lim_{k\to\infty} R_k(u,v) = \lim_{k\to\infty} \beta \frac{1-(1-\beta|H(u,v)|^2)^k}{1-(1-\beta|H(u,v)|^2)} H^*(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & H(u,v) \neq 0 \\ 0 & H(u,v) = 0 \end{cases}$$

Figure 1: Residual error versus number of iterations for the iterative LS algorithm; 1D motion blur over 8 pixels, no noise.

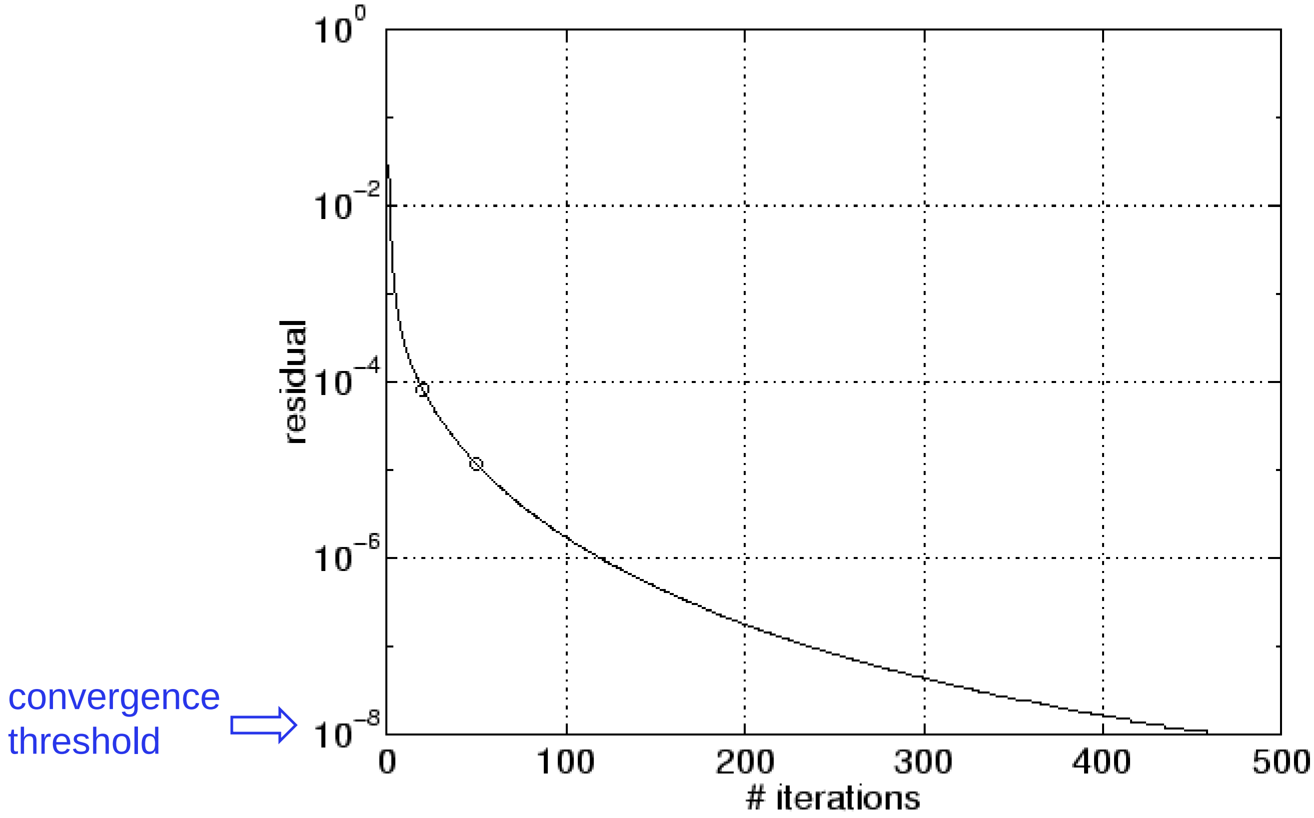


Fig. 2(a): (l) 1D motion blur over 8 pixels; (r) iterative LS restoration, k=20, ISNR=4.03 dB.

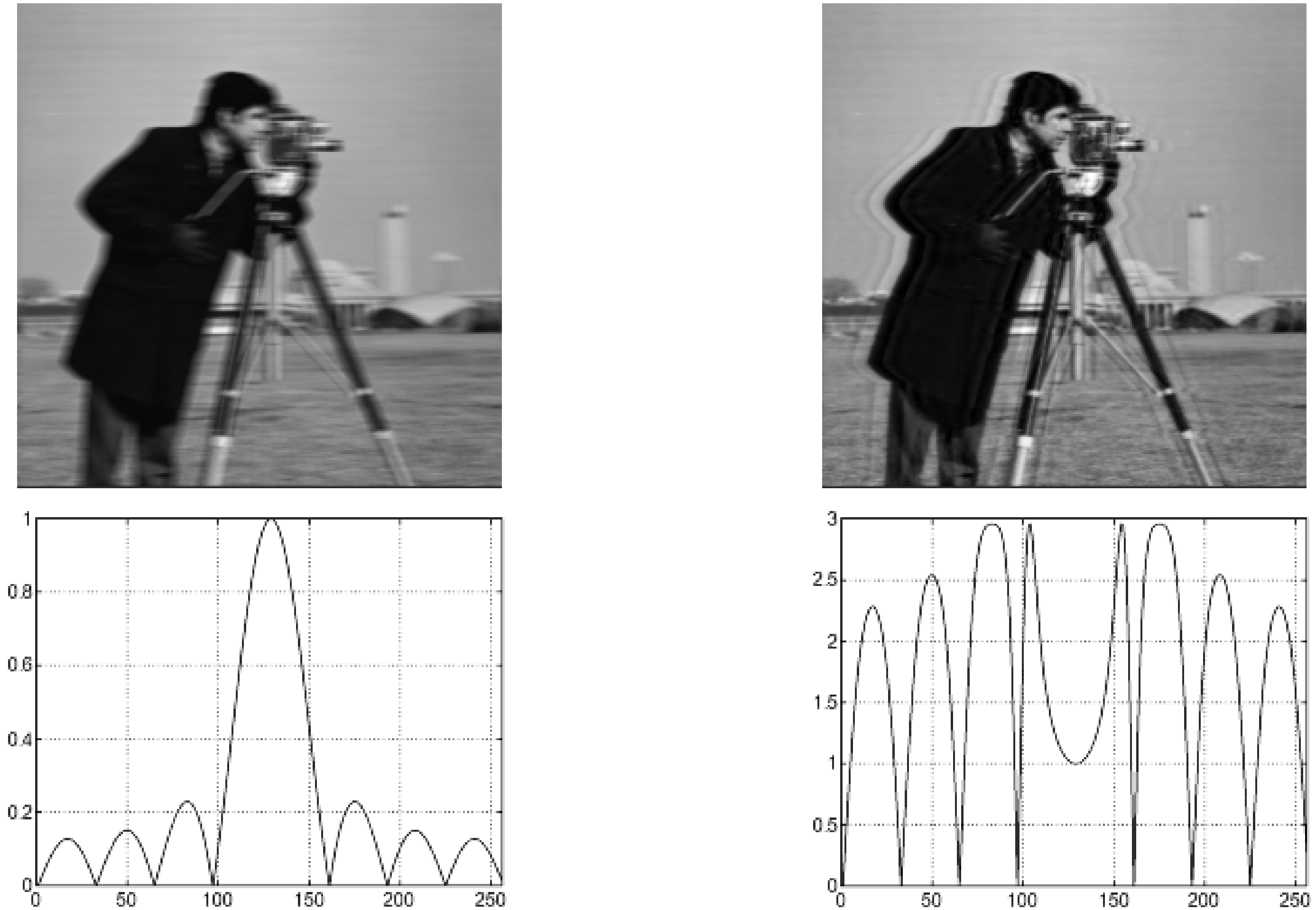


Fig. 2(b): iterative LS restorations: (l) k=50, ISNR=6.22 dB; (r) k=465, ISNR=11.58 dB.

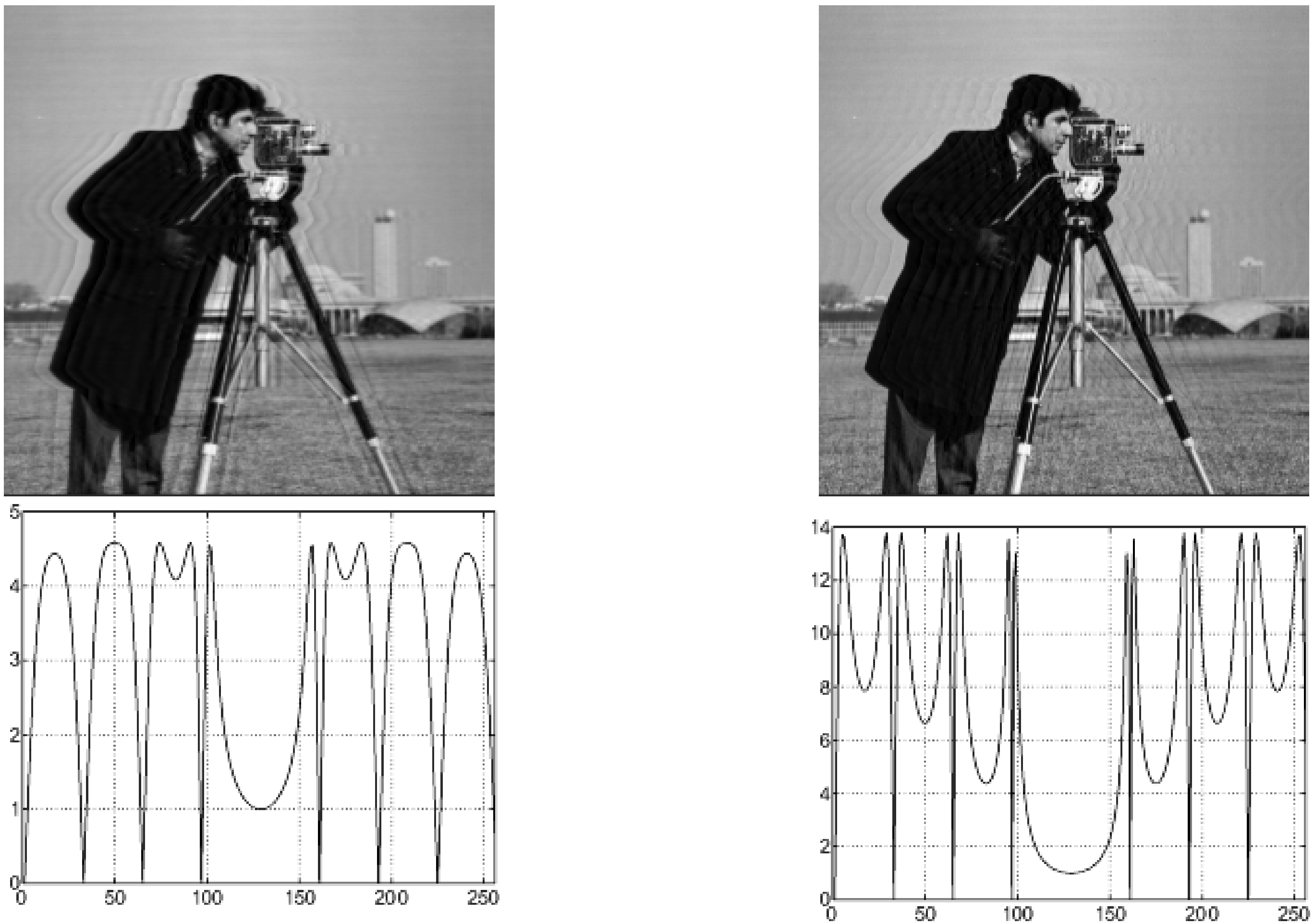
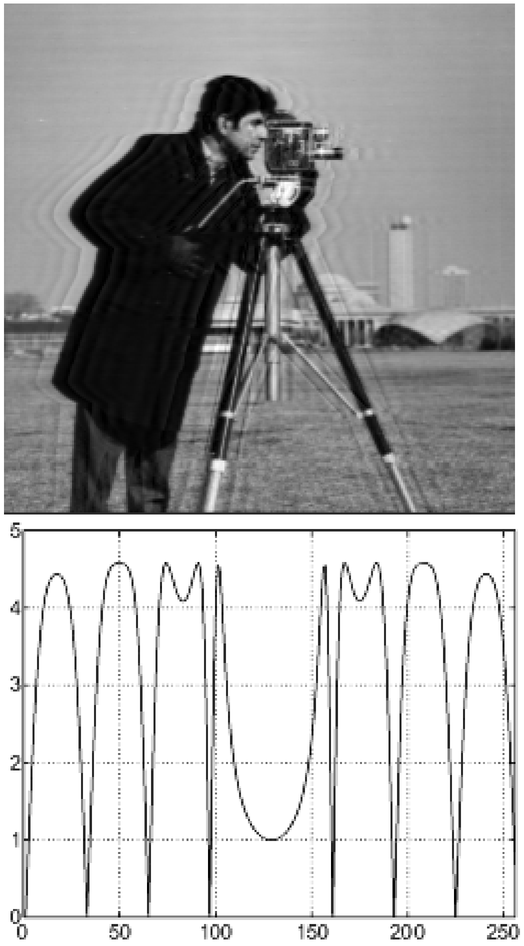
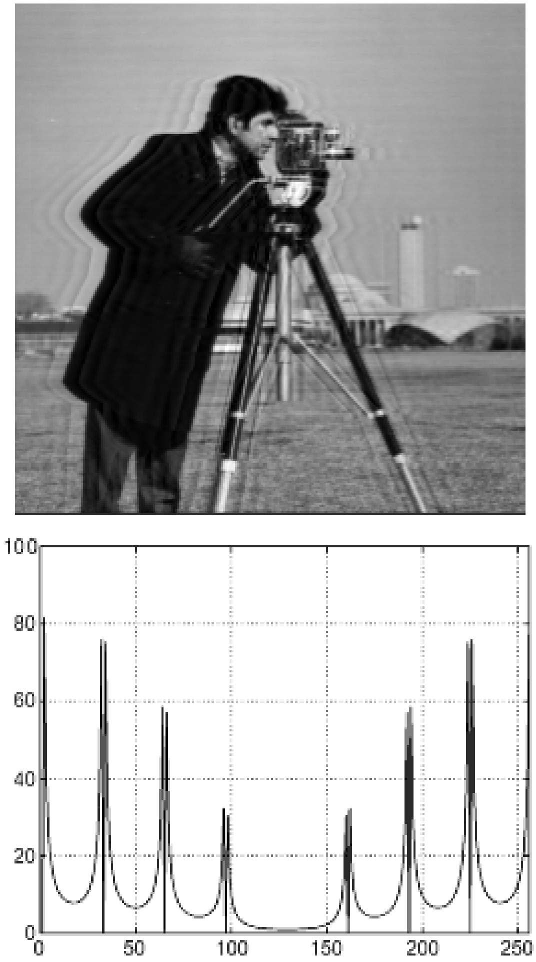


Fig. 2(c): iterative LS restorations: (l) k=465, ISNR=11.58 dB; (r) direct inverse, ISNR=15.50 dB.





Ringing Artifacts

impulse response of overall system → s_{all}; *impulse response of restoration filter* → r

$$s_{all}(i,j) = h(i,j) * r(i,j)$$

restored image → $\hat{f}$; *original image* → f

$$\hat{f}(i,j) = s_{all}(i,j) * f(i,j)$$

Ideally

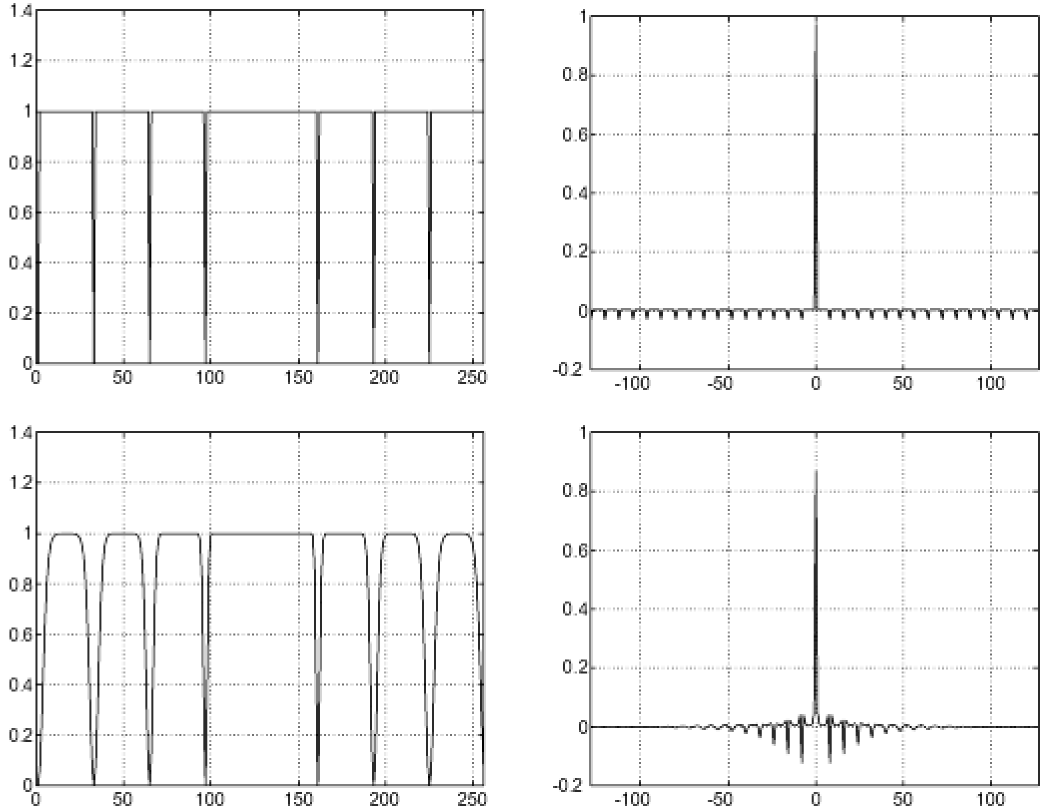
discrete impulse → δ

$$s_{all}(i,j) = \delta(i,j)$$

Or

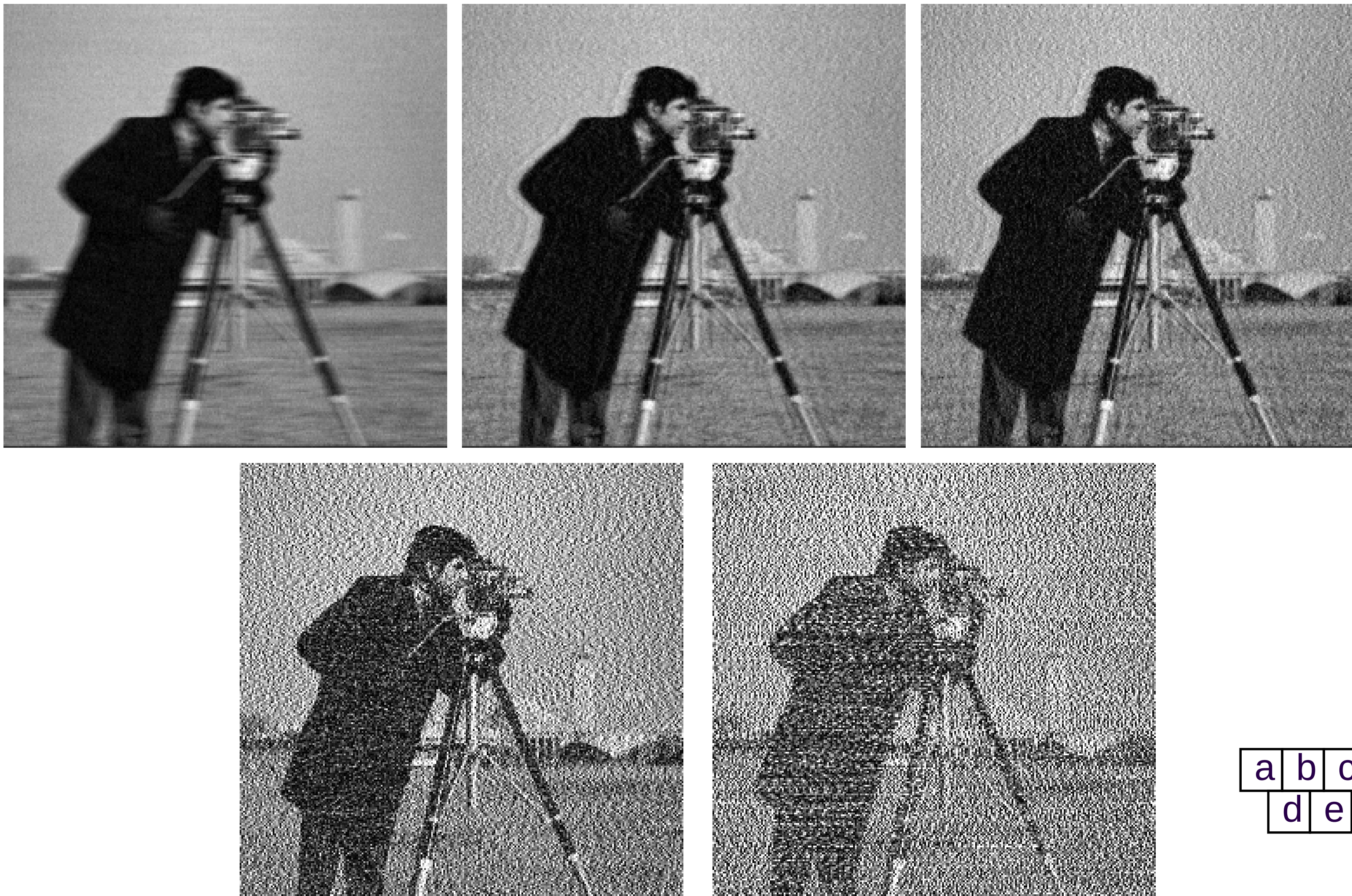
$$S_{all}(u,v) = 1, \quad \forall\ (u,v)$$

Fig. 3: 1D motion blur over 8 pixels; (a), (b): $S_{all}(u,0)$ and $s_{all}(i,0)$ for the direct inverse filter; (c) and (d): $S_{all}(u,0)$ and $s_{all}(i,0)$ for the iterative LS restoration algorithm.



a	b
c	d

Fig. 4: (a) 1D motion blur over 8 pixels, BSNR=20dB; (b)-(d) iterative LS restorations: (b) k=20, ISNR=1.83 dB; (c) k=50, ISNR=-0.30 dB; (d) k=1376, ISNR=-9.06 dB; (e) direct inverse, ISNR=-12.09 dB.



a	b	c
	d	e

Constrained Least Squares (CLS) Iteration

$$\Phi(\mathbf{f}) = \frac{1}{2}\nabla_{\mathbf{f}}\,(\|\mathbf{y}-\mathbf{H}\mathbf{f}\|^2 + \alpha\|\mathbf{C}\mathbf{f}\|^2)$$

$$\mathbf{f}_{\mathbf{k}+\mathbf{1}} = \beta\,\mathbf{H}^{\mathbf{T}}\mathbf{y} + (\mathbf{I} - \beta(\mathbf{H}^{\mathbf{T}}\mathbf{H} + \alpha\,\mathbf{C}^{\mathbf{T}}\mathbf{C}))\mathbf{f}_{\mathbf{k}}$$

Frequency Domain Iteration (H, C block circulant)

$$F_{k+1}(u,v) = \beta H^*(u,v)Y(u,v) + \left(1-\beta\left(|H(u,v)|^2 + \alpha|C(u,v)|^2\right)\right)F_k(u,v)$$

Convergence

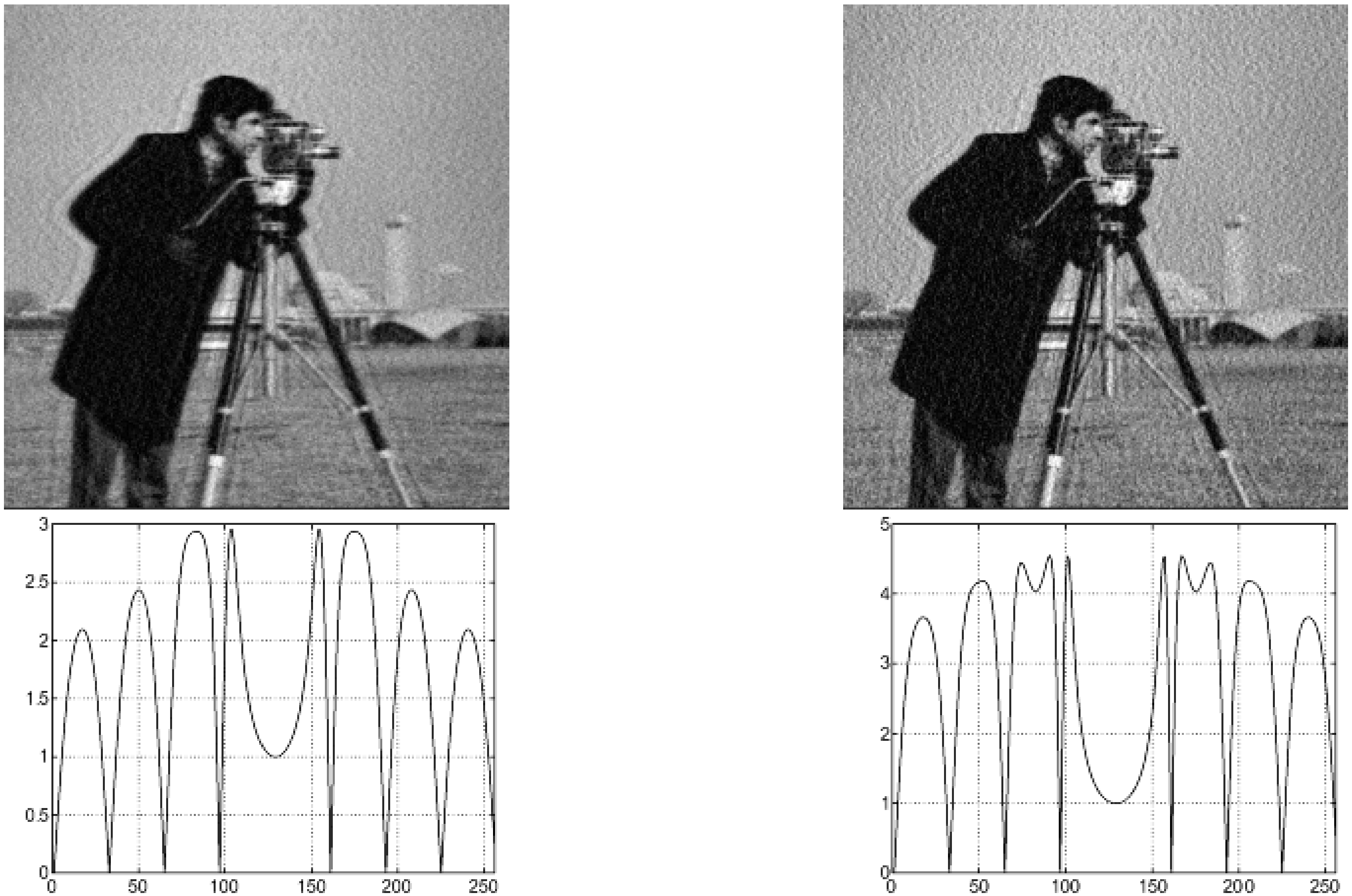
$$R_k(u,v) = \beta \sum_{l=0}^{k-1} \left(1 - \beta\left(|H(u,v)|^2 + \alpha |C(u,v)|^2\right)\right)^l H^*(u,v)$$

sufficient condition for convergence

$$\left|1 - \beta\left(|H(u,v)|^2 + \alpha |C(u,v)|^2\right)\right| < 1$$

$$\lim_{k \to \infty} R_k(u,v) = \begin{cases} \dfrac{H^*(u,v)}{|H(u,v)|^2 + \alpha |C(u,v)|^2} & |H(u,v)|^2 + \alpha |C(u,v)|^2 \neq 0 \\ 0 & |H(u,v)|^2 + \alpha |C(u,v)|^2 = 0 \end{cases}$$

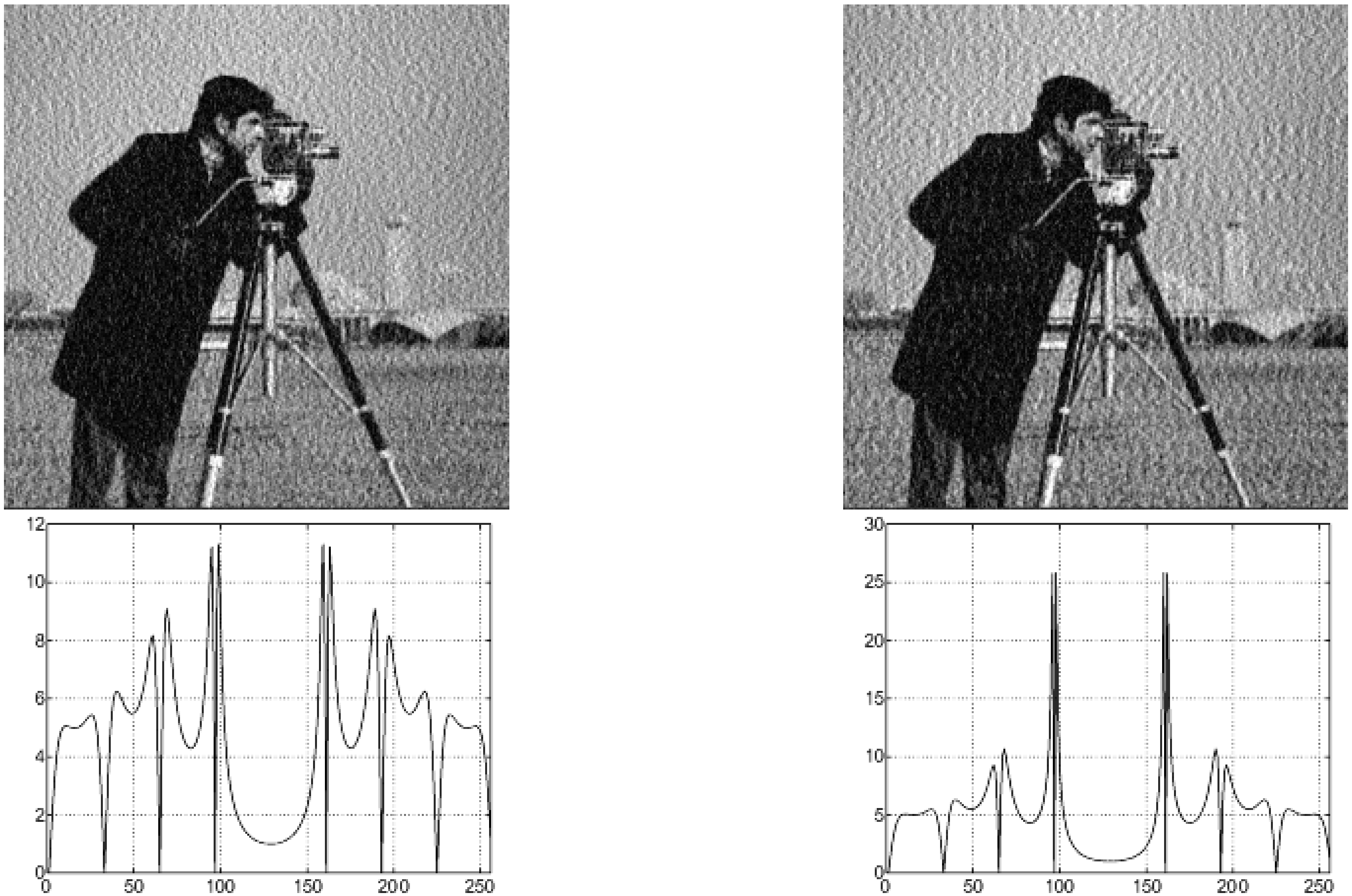
Fig. 5: Restorations of a noisy-blurred image (1D motion blur over 8 pixels, BSNR=20dB) and corresponding $|H(u,0)|$; (a)-(b) iterative CLS restorations, with C a 2D Laplacian and α=0.01: (a) k=20, ISNR=2.12 dB; (b) k=50, ISNR=0.98 dB.



a	b

Fig. 5: Restorations of a noisy-blurred image (1D motion blur over 8 pixels, BSNR=20dB) and corresponding $|H(u,0)|$; (b) iterative CLS restoration with C a 2D Laplacian, α=0.01, k=330, ISNR=-1.01 dB; (d) direct CLS

restoration, ISNR=-1.64 dB.

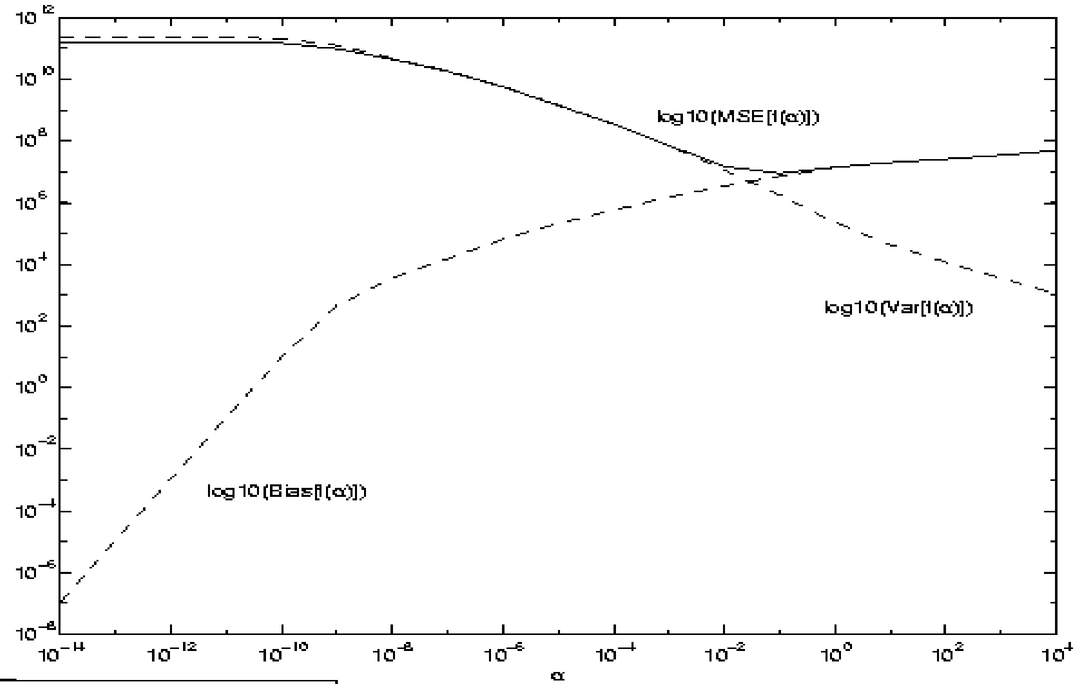


c	d

Effect of Regularization Parameter in CLS Algorithm

$$E\,\|\mathbf{f}(\alpha)-\mathbf{f}\|^2 =$$

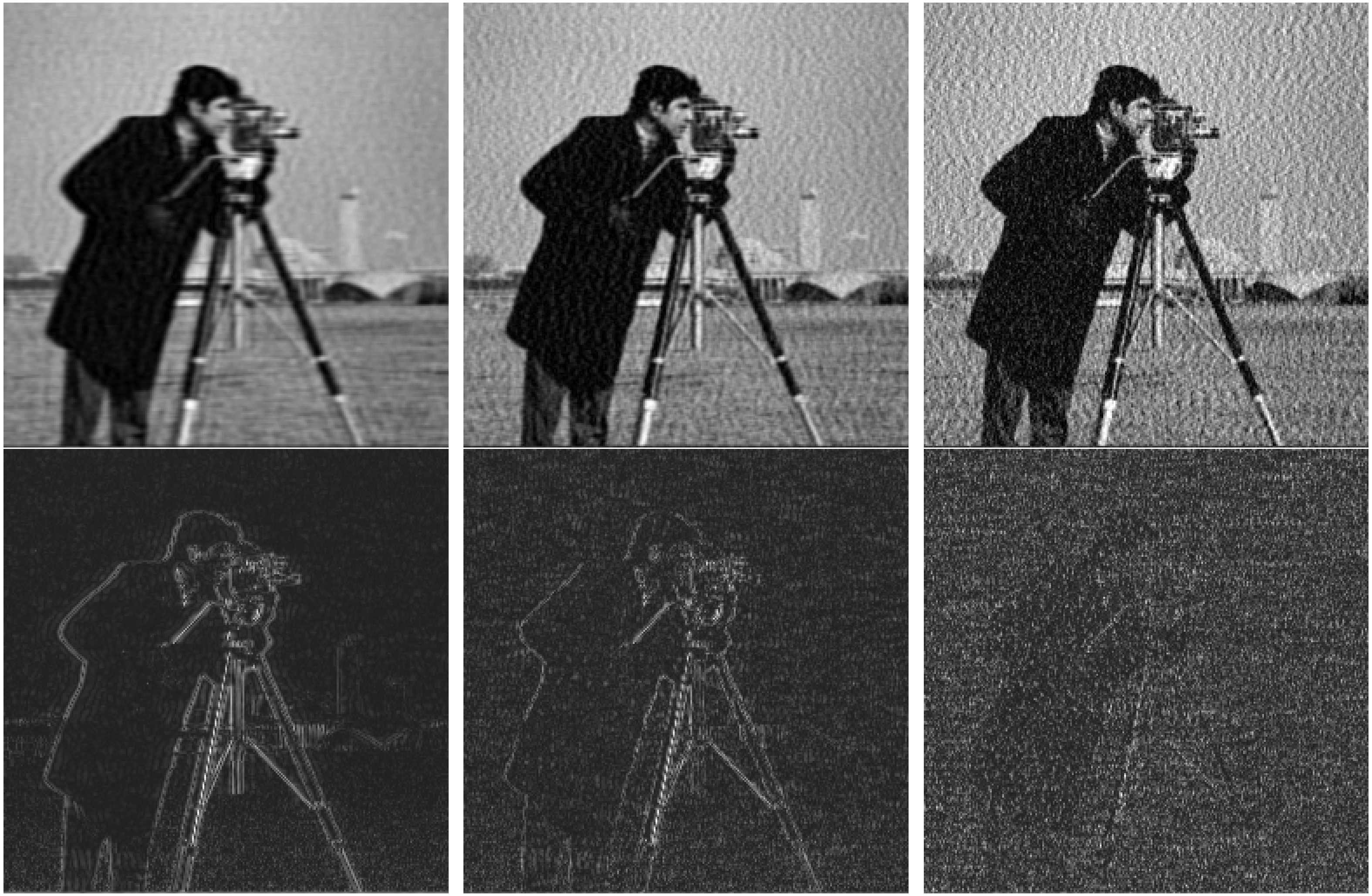
$$Bias\,(\mathbf{f}(\alpha)) + Var\,(\mathbf{f}(\alpha))$$



$$Var(\alpha)=\sigma_n^2\sum_{u=0}^{M}\sum_{v=0}^{N}\frac{|H(u,v)|^2}{\left(|H(u,v)|^2+\alpha|C(u,v)|^2\right)^2}$$

$$Bias\,(\alpha)=\sigma_n^2\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\frac{|F(u,v)|^2\alpha^2|C(u,v)|^4}{\left(|H(u,v)|^2+\alpha|C(u,v)|^2\right)^2}$$

Fig. 6: Direct CLS restorations of a noisy-blurred image (1D motion blur over 8 pixels, BSNR=20dB) with α equal to (a) 1, (b) 01., (c) 0.01; (d)-(f) corresponding |original-restored| linearly mapped to the [32,255] range.



a	b	c
d	e	f

Spatially Adaptive Constrained Least Squares Iteration

$$\Phi(\mathbf{f}) = \frac{1}{2}\nabla_{\mathbf{f}}\left(\|\mathbf{y}-\mathbf{Hf}\|^{\mathbf{2}}_{\mathbf{W_1}} + \alpha\ \|\mathbf{Cf}\|^{2}_{\mathbf{W_2}}\right)$$

$$\mathbf{f_{k+1}} = \beta\,\mathbf{H^T W_1^T W_1 y} + (\mathbf{I} - \beta(\mathbf{H^T W_1^T W_1 H} + \alpha\,\mathbf{C^T W_2^T W_2 C}))\mathbf{f_k}$$

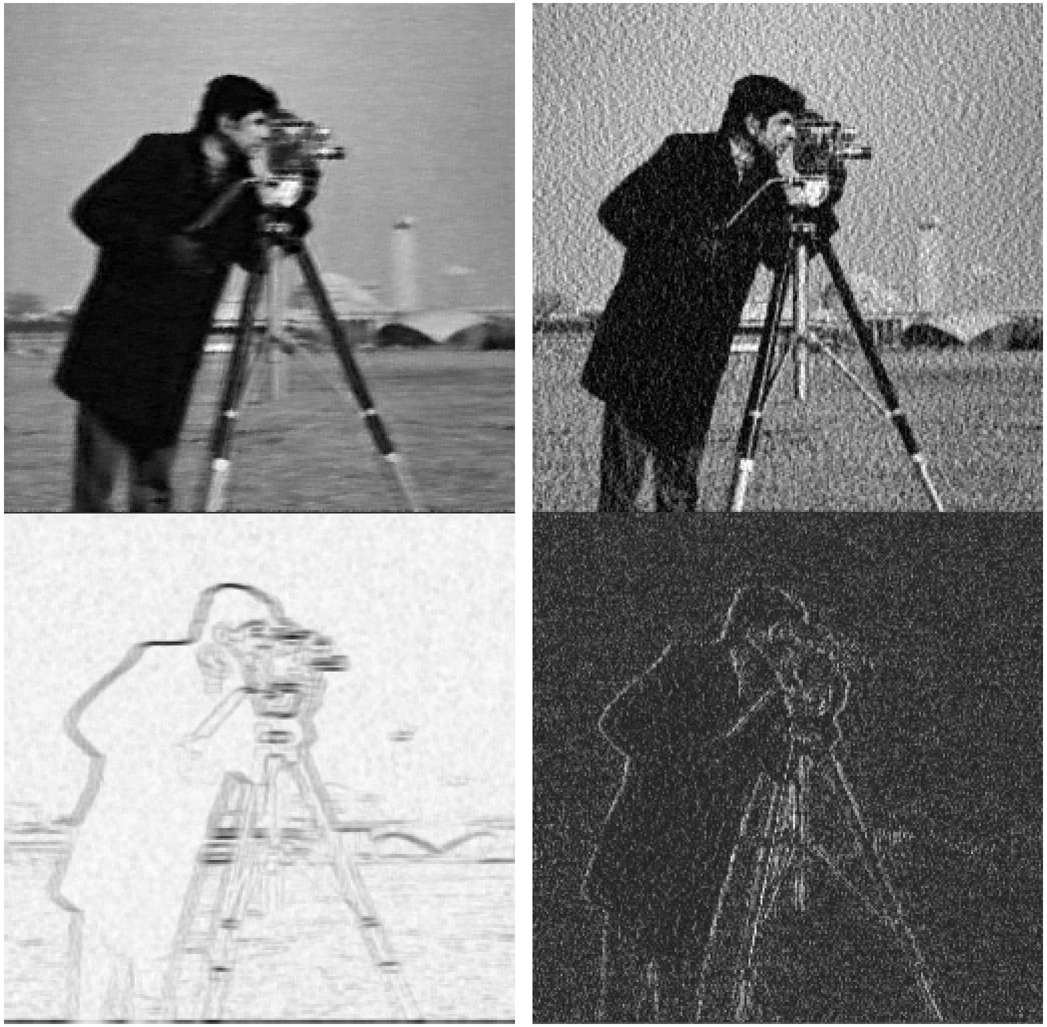
Choice of weights

$\mathbf{W_1} = \mathbf{1} - \mathbf{W_2},\ \ \mathbf{W_2} = \mathbf{V}$ (the visibility matrix)

$\mathbf{V} \approx 1/(\sigma^2)$ (measure of the local activity)

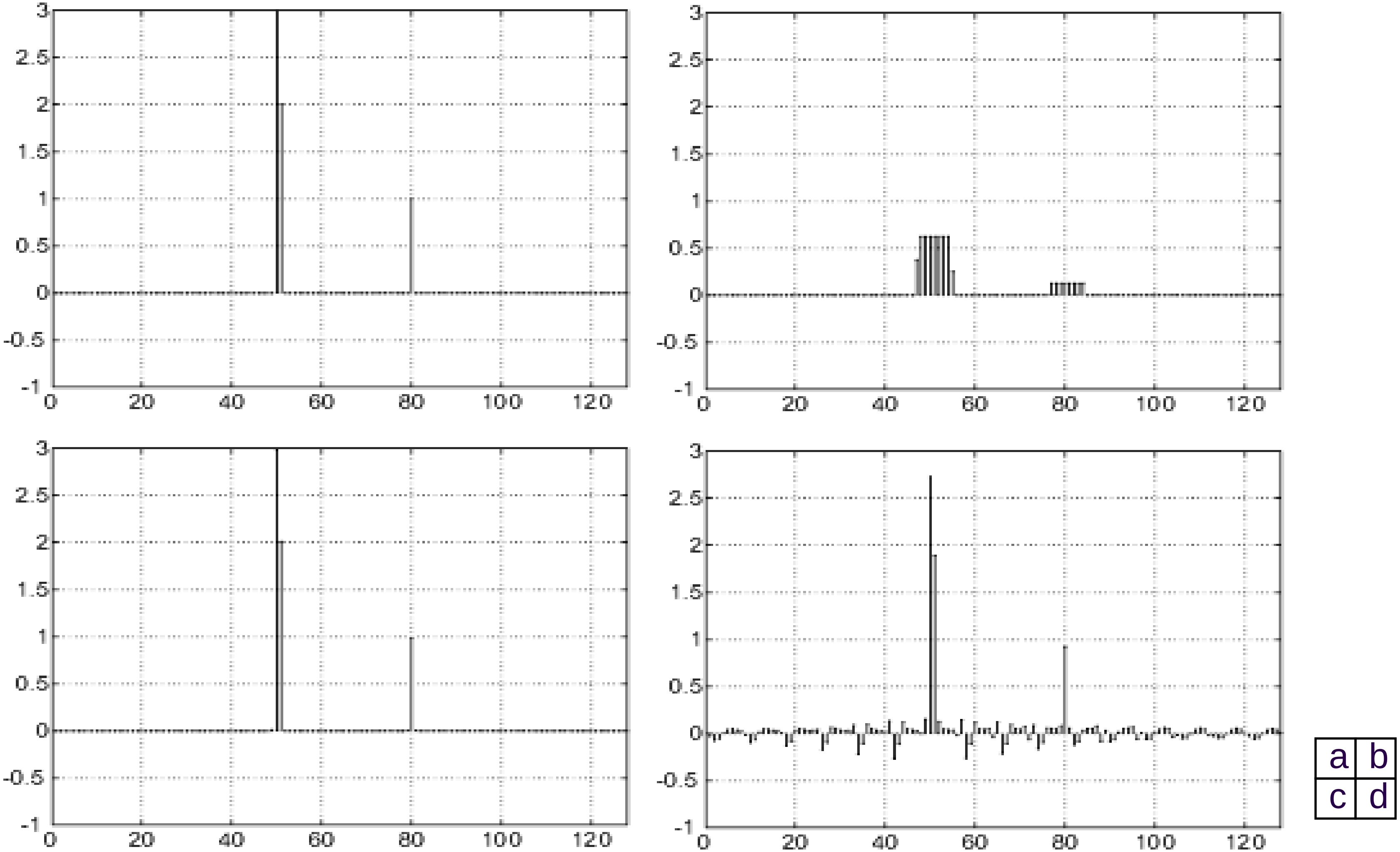
Fig. 7: Restorations of a noisy-blurred image (1D motion blur over 8 pixels, BSNR=20dB); (a) iterative adaptive CLS; (b) iterative CLS; (c) entries of visibility matrix linearly mapped to the [32-255] range; (d) |fig. a - fig. b|

linearly mapped to the [32,255] range.



a	b
c	d

Fig. 7: (a) original signal; (b) blurred signal by 1D motion blur over 8 samples; (c) iterative LS with positivity constraint; (d) iterative LS without positivity constraint.



Wiener Filter

$$\hat{\mathbf{f}} = \arg\{\min_{\mathbf{f}}\{E\{(\mathbf{f}-\hat{\mathbf{f}})^{\mathbf{T}}(\mathbf{f}-\hat{\mathbf{f}})\}\}$$

$$\hat{\mathbf{f}} = \mathbf{R_{ff}}\mathbf{H}^{\mathbf{T}}(\mathbf{H}\mathbf{R_{ff}}\mathbf{H}^{\mathbf{T}} + \mathbf{R_{nn}})^{-1}\mathbf{y}$$

f and **n** uncorrelated and zero mean

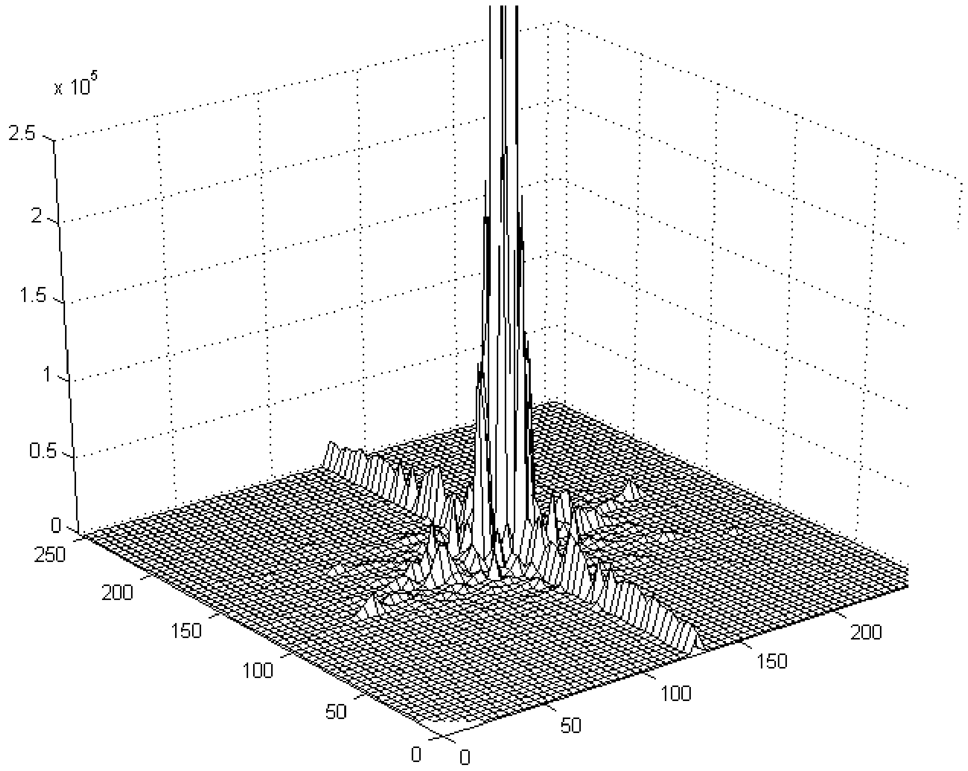
Image autocorrelation

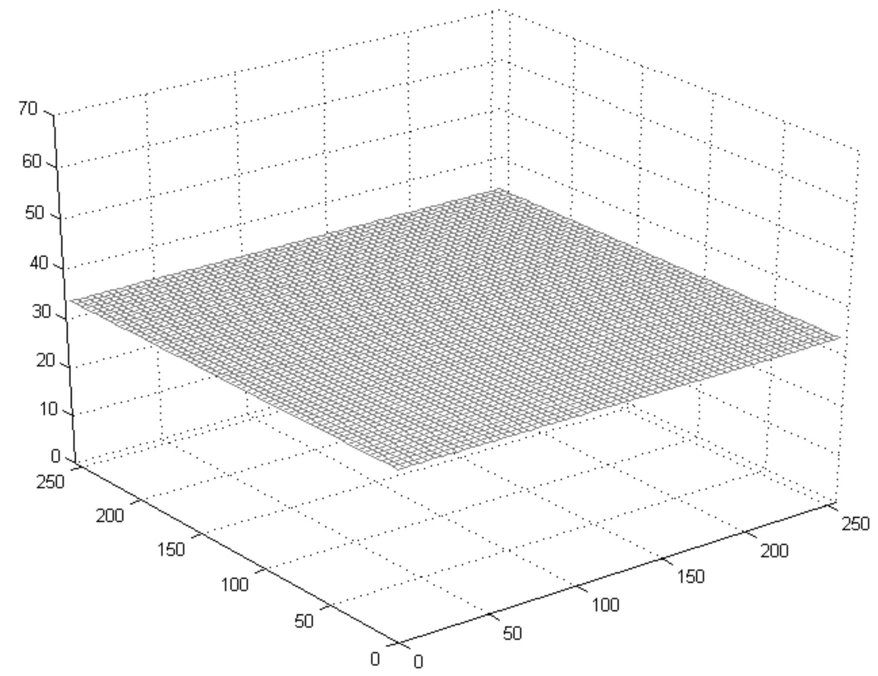
Noise autocorrelation

Frequency Domain (all matrices block circulant)

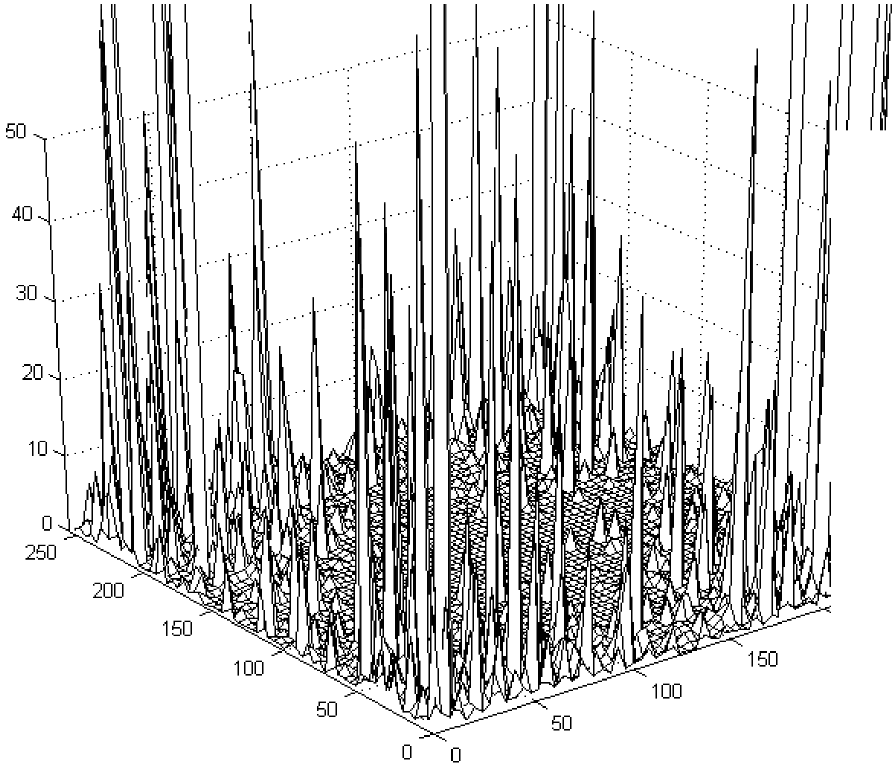
$$\hat{F}(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + S_{nn}(u,v)/S_{ff}(u,v)}Y(u,v)$$

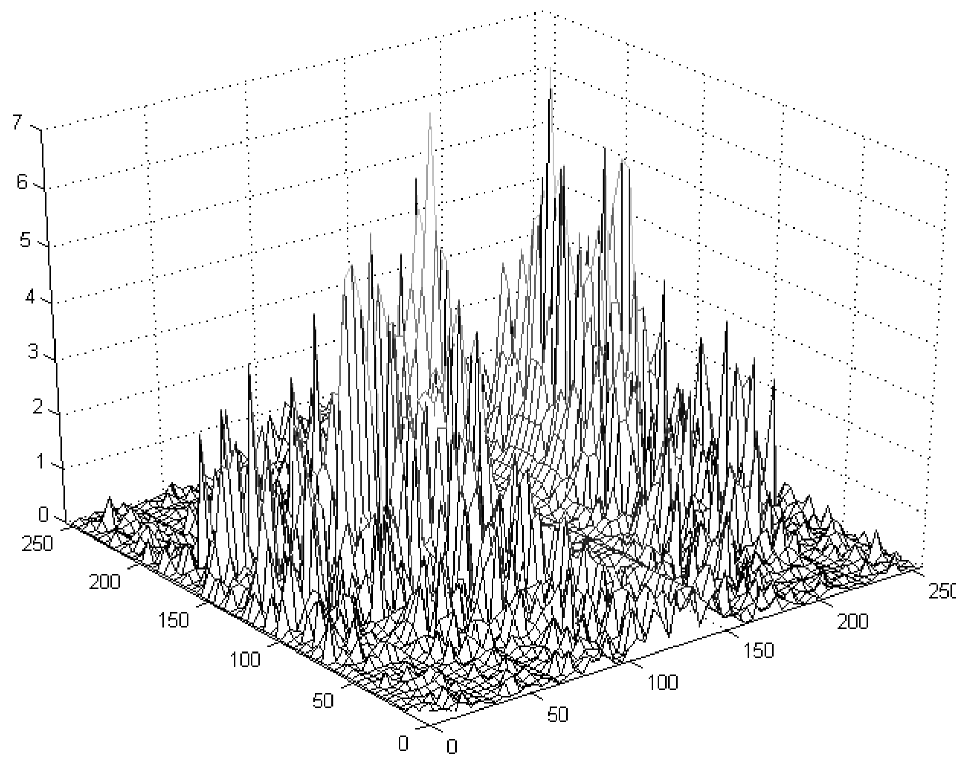
Power spectral density of original image $S_{ff}(u,v)$ and noise $S_{nn}(u,v)$



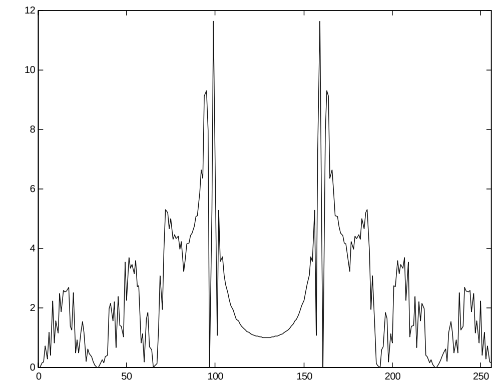


(l) Stabilizing term $S_{nn}(u,v)/S_{ff}(u,v)$ and (r) magnitude of the frequency response of the Wiener filter

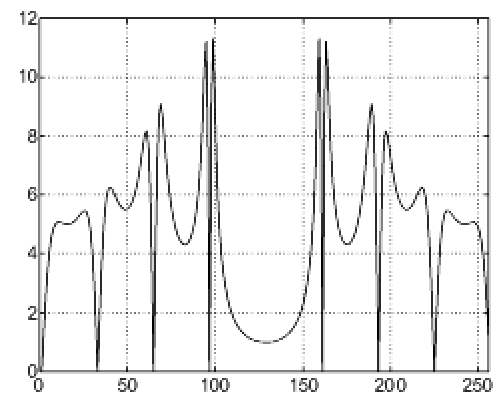




$|H(u,0)|$



Wiener filter



Noisy-blurred image;
1D motion blur over 8 pixels;
BSNR=20dB.

Wiener restoration;
S_ff from original image,
S_nn ideal, ISNR=3.93 dB

Iterative CLS restoration
with C a 2D Laplacian,
=0.01, k=330, ISNR=-1.01 dB.

Bayesian Framework: $p(\mathbf{f}\,|\,\mathbf{y}) = \dfrac{p(\mathbf{y}\,|\,\mathbf{f})\,p(\mathbf{f})}{p(\mathbf{y})}$

noise model

image model

$$\mathbf{f}_{MAP} = \arg\max_{\mathbf{f}} \; p(\mathbf{f} / \mathbf{y})$$

For Gaussian Image and Noise Models

$$p(\mathbf{f}) = \frac{1}{(2\pi)^{N/2}\,|\mathbf{C}_{\mathbf{f}}|^{1/2}}\, e^{-\frac{1}{2}\mathbf{f}^{T}\mathbf{C}_{\mathbf{ff}}^{-1}\mathbf{f}}$$

$$p(\mathbf{n}) = p(\mathbf{y}\,|\,\mathbf{f}) = \frac{1}{(2\pi)^{N/2}\,|\mathbf{C}_{\mathbf{nn}}|^{1/2}}\, e^{-\frac{1}{2}(\mathbf{y}-\mathbf{Hf})^{T}\mathbf{C}_{\mathbf{nn}}^{-1}(\mathbf{y}-\mathbf{Hf})}$$

$$\mathbf{f}_{MAP} = \arg\min_{\mathbf{f}} \left\{ (\mathbf{y}-\mathbf{Hf})^{T}\mathbf{C}_{\mathbf{nn}}^{-1}(\mathbf{y}-\mathbf{Hf}) + \mathbf{f}^{T}\mathbf{C}_{\mathbf{ff}}^{-1}\mathbf{f} \right\} \Longrightarrow$$

$$\left(\mathbf{H}^{T}\mathbf{C}_{\mathbf{nn}}^{-1}\mathbf{H} + \mathbf{C}_{\mathbf{ff}}^{-1}\right)\mathbf{f}_{MAP} = \mathbf{H}^{T}\mathbf{C}_{\mathbf{nn}}^{-1}\mathbf{y}$$

Relationship between Wiener and MAP estimates for Gaussian case

since

$$\mathbf{C_{ff}H^T}\left[\mathbf{HC_{ff}H^T}+\mathbf{C_{nn}}\right]^{-1}=\left[\mathbf{H^TC_{nn}^{-1}H}+\mathbf{C_{ff}}^{-1}\right]^{-1}\mathbf{H^TC_{nn}^{-1}}$$

$$\Longrightarrow \mathbf{f}_{\text{Wiener}}=\mathbf{f}_{\text{MAP}}$$

Relationship between Wiener and CLS

if $$\left[\mathbf{H^TC_{nn}^{-1}H}+\mathbf{C_{ff}}^{-1}\right]^{-1}\mathbf{H^TC_{nn}^{-1}}=\left[\mathbf{H^TH}+\alpha\,\mathbf{C^TC}\right]\mathbf{H^T}$$

or $\mathbf{C_{ff}}=\sigma_{\mathbf{n}}^2\left(\alpha\mathbf{C^TC}\right)^{-1}$ for $\mathbf{C_{nn}}=\sigma_{\mathbf{n}}^2\,\mathbf{I}$

$$\Longrightarrow \mathbf{f}_{\text{Wiener}}=\mathbf{f}_{\text{CLS}}$$

Topics not Covered

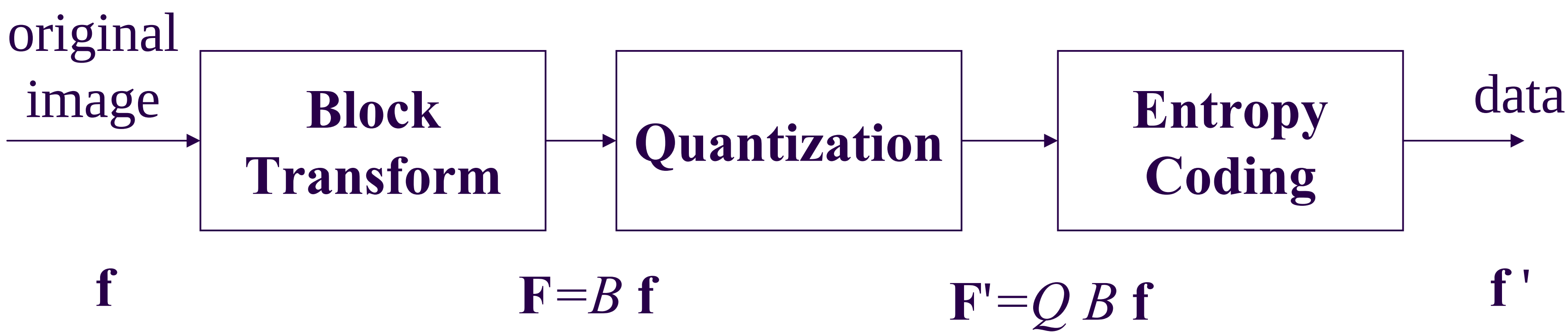
- Use of robust statistics
- Kalman filters
- Iteration adaptive algorithms
- Use of wavelets
- Multi-channel restoration
- Partially-known degradations (TLS approach)
- Signal-dependent noise models
- Non-linear degradation model
- Blind image restoration
- Video restoration

Recovery of Compressed Images and Video

- Removal of Blocking Artifacts
  - enhancement techniques
  - restoration techniques

- Removal of Additional Quantization Artifacts in Compressed Video

Transform Based Coder

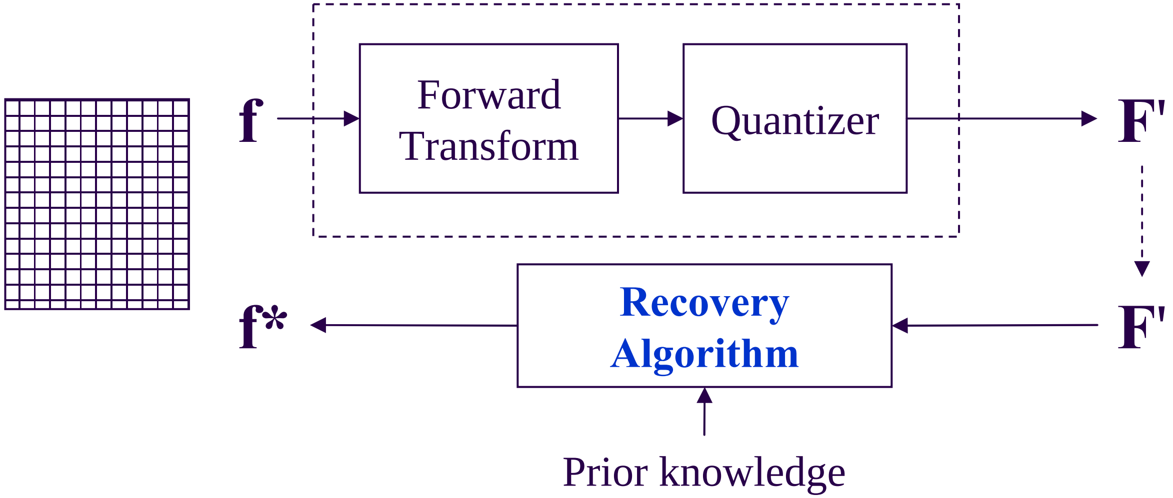


- **"Conventional" decoder:**

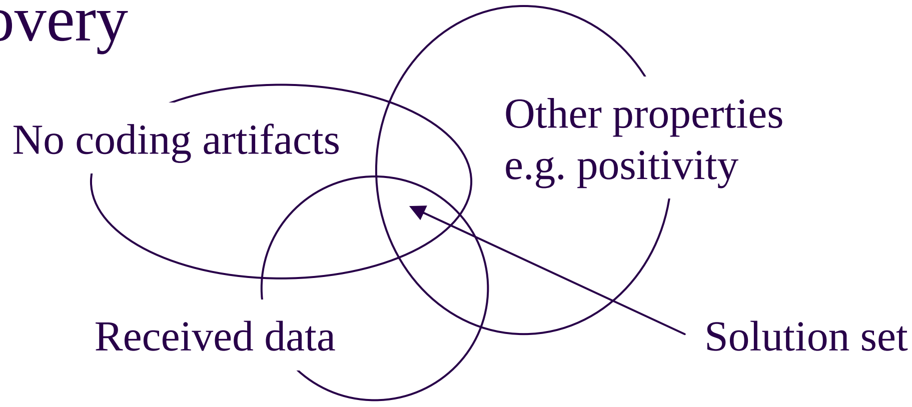
$$\mathbf{f}' = B^t \mathbf{F}'$$

Proposed Decoding Approach

- Image Decoding ⇔ Image Recovery



- Set Theoretic Recovery Principle



Projection onto Data Set C_d

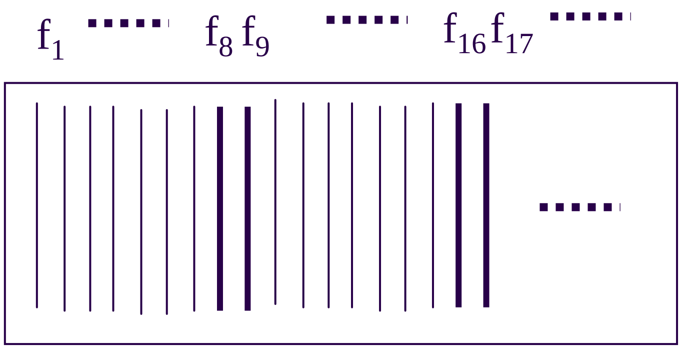
- $C_d \equiv \{\mathbf{f}: QB\,\mathbf{f} = \mathbf{F}'\}$
  or
  $C_d \equiv \{\mathbf{f}: \mathbf{F}_n^{min} \le (B\,\mathbf{f})_n \le \mathbf{F}_n^{max}\}$
  - $\mathbf{F}_n^{min}$, $\mathbf{F}_n^{max}$ determined by quantizer
- Easily verified that $\mathbf{F}' \in C_d$
- $P_d\mathbf{f} = B^{-1}\mathbf{F}$
  - where

$$\mathbf{F}_n = \begin{cases} \mathbf{F}_n^{min} & \text{if} \quad (B\mathbf{f})_n < \mathbf{F}_n^{min} \\ \mathbf{F}_n^{max} & \text{if} \quad (B\mathbf{f})_n < \mathbf{F}_n^{max} \\ (B\mathbf{f})_n & \text{if} \quad \mathbf{F}_n^{min} \le (B\mathbf{f})_n \le \mathbf{F}_n^{max} \end{cases}$$

Spatial Smoothness

- $C_s \equiv \{\ \mathbf{f}\text{: } \mathbf{f} \text{ is smooth in the block boundaries }\}$

- Between Block Discontinuity



- Constraint Set

$$C_s \equiv \{\ \mathbf{f}\text{: } \| W Q\mathbf{f} \| \leq E\ \}, \text{ with } \mathbf{Qf} = \begin{bmatrix} \mathbf{f_8} - \mathbf{f_9} \\ \mathbf{f_{16}} - \mathbf{f_{17}} \\ \mathbf{f_{24}} - \mathbf{f_{25}} \\ \vdots \end{bmatrix}$$

P_s:Projection onto C_s

- $\mathbf{f}=\{\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_N\}$, $\quad \tilde{\mathbf{f}} = P_s\mathbf{f} = \left\{\tilde{\mathbf{f}}_1, \tilde{\mathbf{f}}_2, \cdots, \tilde{\mathbf{f}}_N\right\}$
- define

$$\mathbf{x} = \begin{bmatrix} \mathbf{f}_8 \\ \mathbf{f}_{16} \\ \vdots \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} \mathbf{f}_9 \\ \mathbf{f}_{17} \\ \vdots \end{bmatrix} \quad \tilde{\mathbf{x}} = \begin{bmatrix} \tilde{\mathbf{f}}_8 \\ \tilde{\mathbf{f}}_{16} \\ \vdots \end{bmatrix} \quad \tilde{\mathbf{y}} = \begin{bmatrix} \tilde{\mathbf{f}}_9 \\ \tilde{\mathbf{f}}_{17} \\ \vdots \end{bmatrix}$$

- then

$$\tilde{\mathbf{x}} = \tfrac{1}{2}(\mathbf{x}+\mathbf{y}) + \tfrac{1}{2}\left(I + 2\lambda W^t W\right)^{-1}(\mathbf{x}-\mathbf{y})$$
$$\tilde{\mathbf{y}} = \tfrac{1}{2}(\mathbf{x}+\mathbf{y}) - \tfrac{1}{2}\left(I + 2\lambda W^t W\right)^{-1}(\mathbf{x}-\mathbf{y})$$
$$\tilde{f}_i = f_i, \quad for \quad i \neq 8\cdot k \quad or \quad 8\cdot k+1, \quad k = 1,2,\ldots$$

- λ satisfies

$$\|W(I + 2\lambda W^t W)^{-1} Q\,\mathbf{f}\| = E$$

Estimating *W*

- Principle: ω_i should be proportional to
  - sensitivity of HVS to coding artifacts
  - local correlation

- Visibility of blocking artifact
  - less visible in very bright or very dark areas
  - less visible in intensity transition area, such as texture

- An example:

$$\omega_i = \begin{cases} \dfrac{\sqrt{\mu_i}}{1+\sigma_i} & \textit{if } \mu_i < 128 \\ \dfrac{\sqrt{255-\mu_i}}{1+\sigma_i} & \textit{otherwise} \end{cases}$$

POCS-Based Decoding Algorithm

- Constraint sets:
  - C_d : data consistency
  - C_s : horizontal blocking
  - C_s' : vertical blocking
  - C_r : pixel-intensity range

- Algorithm: $\mathbf{f}_k = P_r P_s' P_s P_d \mathbf{f}_{k-1}$
  - terminate when $\| f_k - f_{k-1} \| \leq \varepsilon$
  - typically 3~5 iterations

Compressed at .29bpp

Reconstructed POCS

Reconstructed alg.1

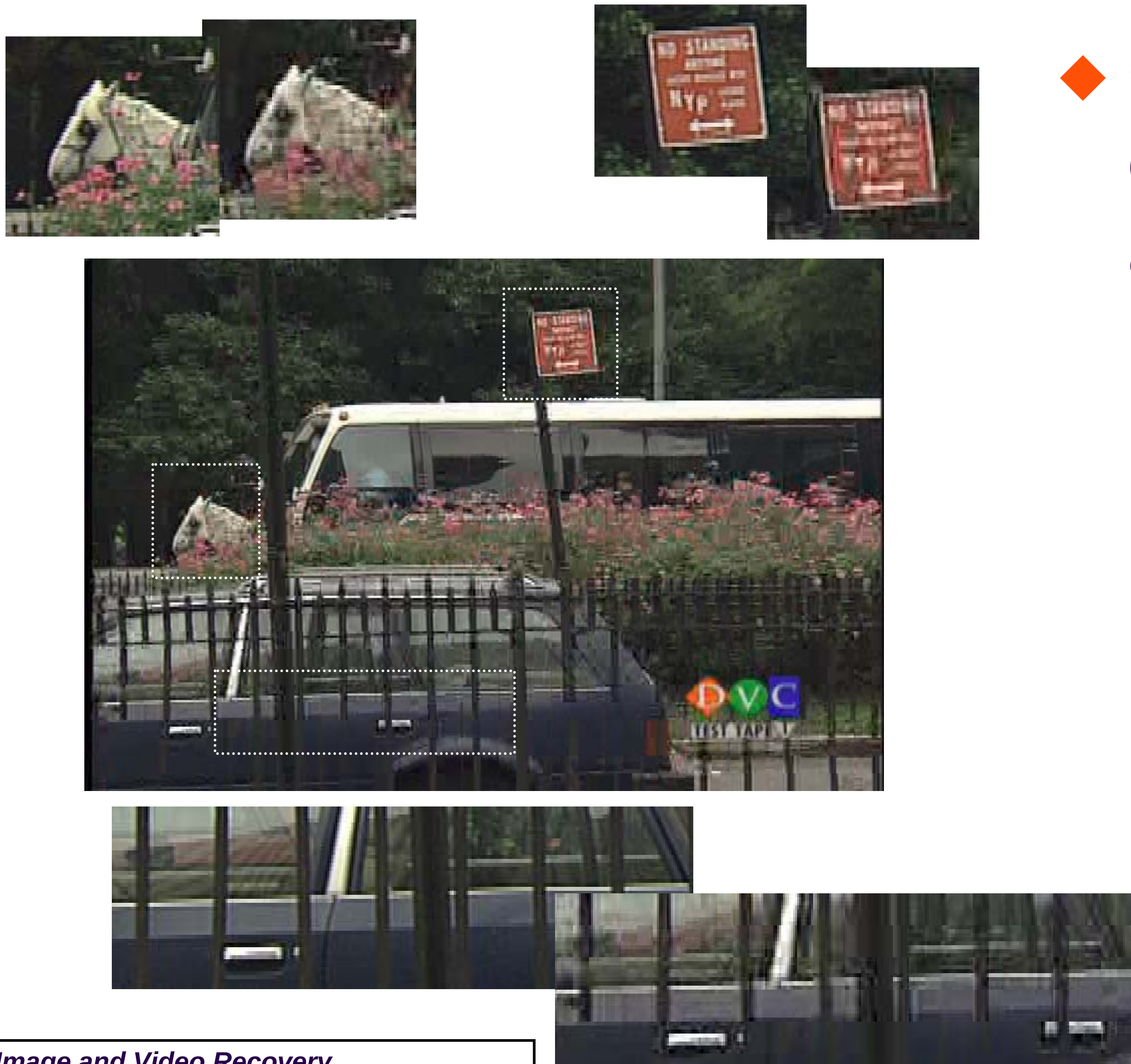
Reconstructed WLS

Reconstructed alg. 2 + P

Reconstructed alg. 3

Experimental Results

Video Recovery

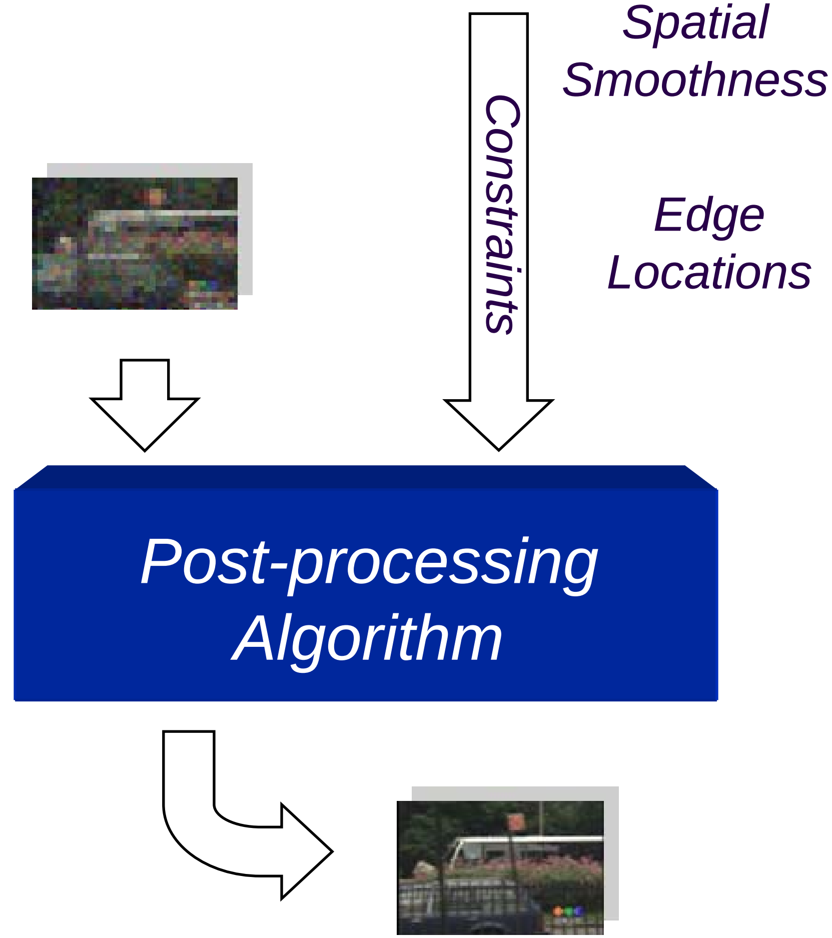


MPEG-2 Encoded at 3.5Mbps

- Current video encoders introduce a variety of artifacts
  - Blocking artifacts dominate lower bitrate applications
  - Ringing artifacts appear as the bitrate increases
  - Visual quality increased by pre- and post-processing

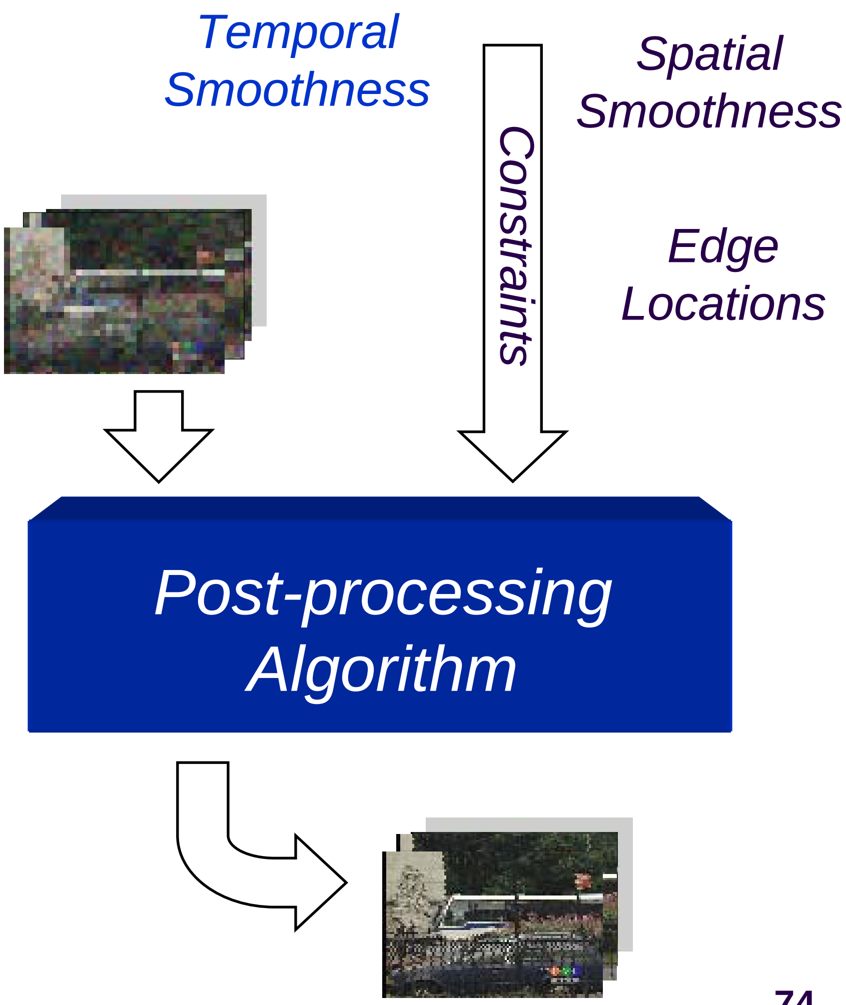
Post-processing

- Post-processing exploits traits of "good" images to reduce coding artifacts
  - Images are generally smooth
  - There are "good" and "bad" edges in the image



Post-processing

- Video post-processing also incorporates properties of a “good” video sequence
  - Motion compensated frames should be similar



Problem Formulation

- Find a recovered image as the minimizer of

$$J(\mathrm{f}_k) = \|\mathrm{f}_k - \mathrm{g}_k\|^2 + \lambda_1 \|Q_1 \mathrm{f}_k\|^2$$

$$+ \lambda_2 \|Q_2 \cdot \mathrm{f}_k\|^2 + \lambda_3 \|\mathrm{f}_k - \mathrm{f}_{mc}\|^2$$

Fidelity to Decoded Image

Measure of Smoothness within Blocks

Measure of Smoothness Between Block

Measure of Temporal Continuity

f_k *can be replaced by a multi-channel vector* $\underline{\mathrm{f}}$

- Iterative solution

$$\underline{f}^{k+1} = \underline{f}^{k} + \alpha\left(\underline{g} - \left((I + \lambda_1 Q_1^T Q_1 + \lambda_2 Q_2^T Q_2)\underline{f}^{k} + 2\lambda_3 \left\|\underline{f} - \underline{f}_{mc}\right\|\right)\right)$$

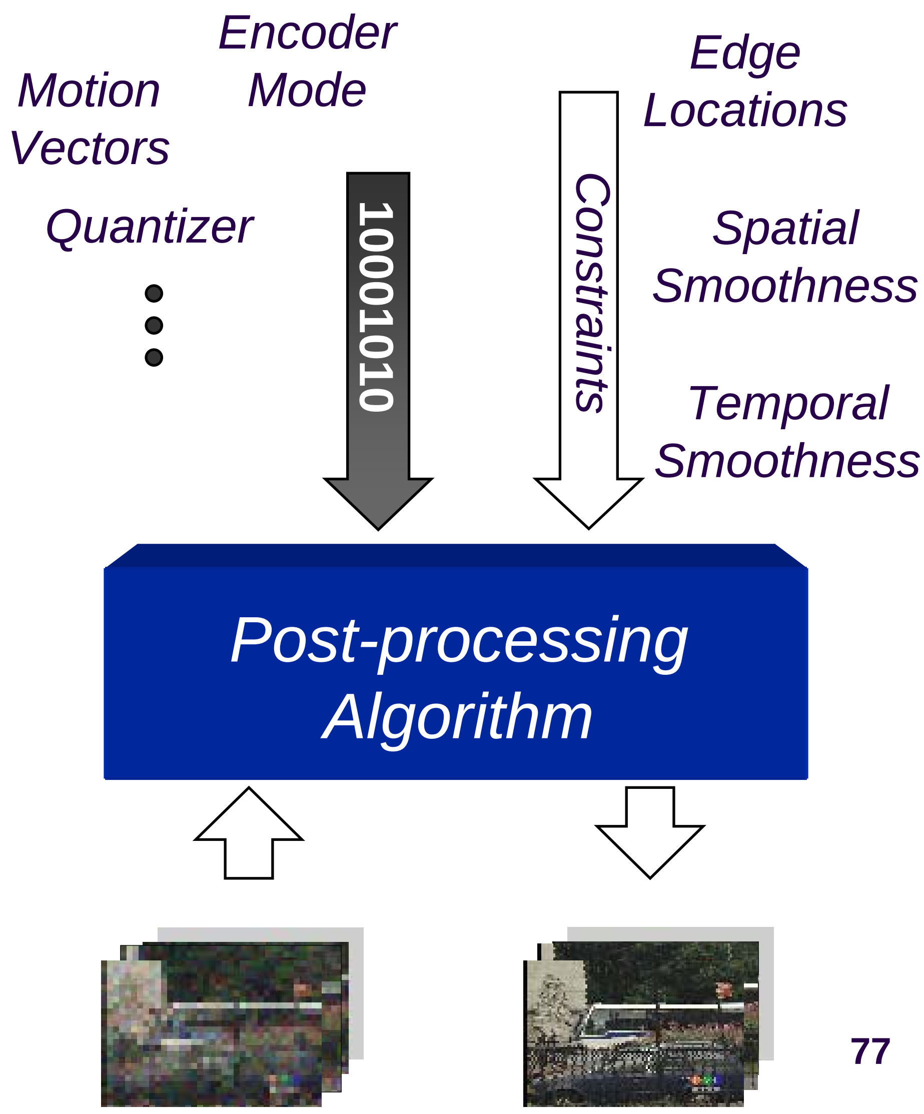
Experimental Results

Compressed (3.5Mbps)

Restored $\lambda_1 = 0.2, \lambda_2 = 0.2, \lambda_3 = 1$

Use Information from Encoder

- Post-processing algorithm can incorporate *all* information available in the compressed bitstream
  - Quantizer step size
  - Encoder Mode Selection
  - Motion Vectors



Constrained Optimization

- We are now considering the post-processing algorithm as a minimization of

Measure of Smoothness within Block

$$J(f_k) = ||Q_1 \cdot f_k||^2 + \lambda_1 ||Q_2 \cdot f_k||^2$$

Measure of Smoothness between Blocks

$$+ \lambda_2 ||f_k - f_{mc}||^2$$

Fidelity to Decoded Image

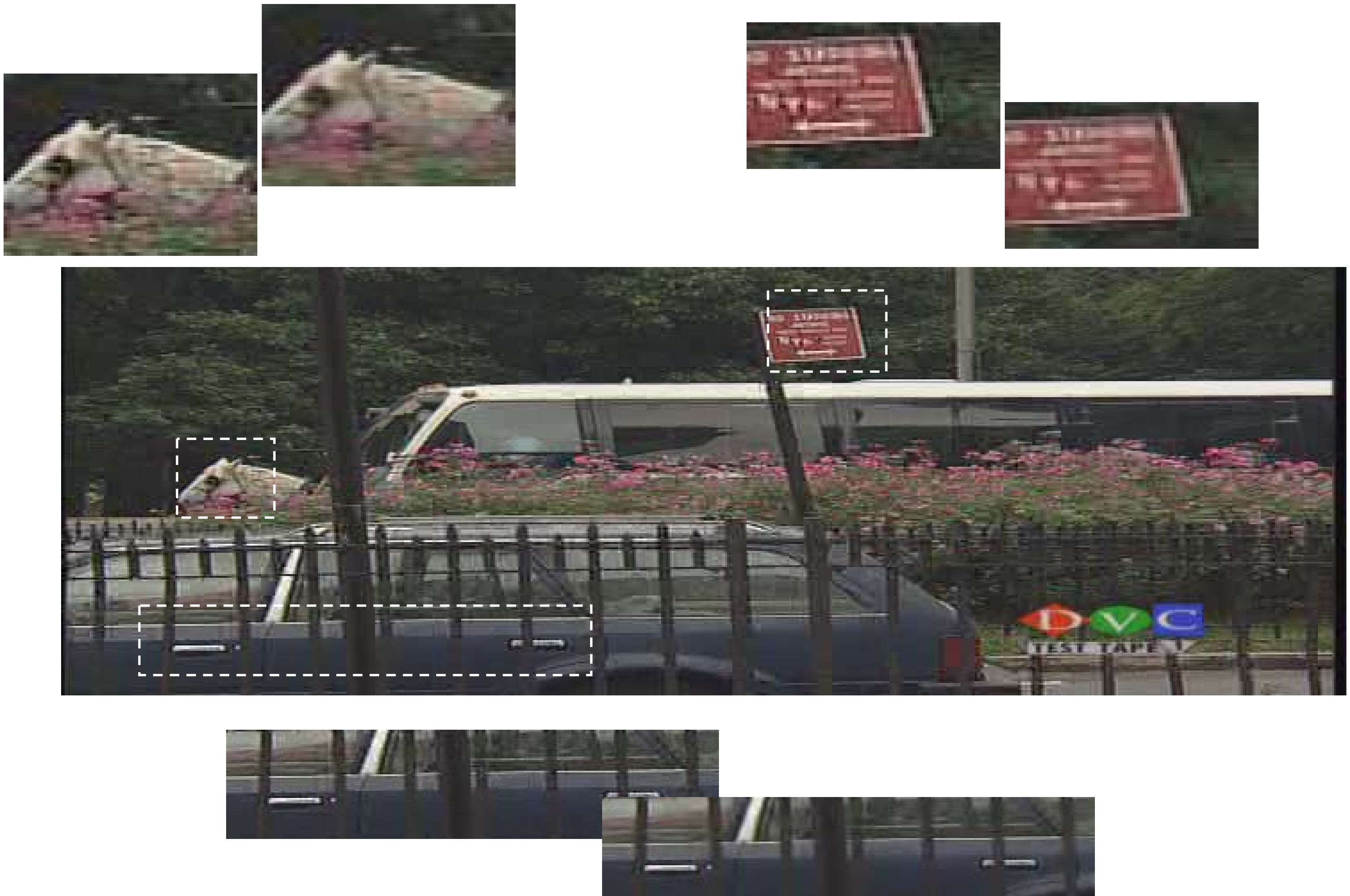
$$s.t.\ \mathrm{DCT}(f_k) \in S_Q$$

Measure of Temporal Continuity -- Motion Vectors are Known

Experimental Results

P-Frame

Experimental Results



P-Frame

Weighted Minimization

- Adding weight matrices to the cost function allows further smoothing adaptation

$$J(\mathrm{f}_k) = \| W_1 \cdot Q_1 \cdot \mathrm{f}_k \|^2$$

Measure of Smoothness within Block

$$+ \lambda_1 \| W_2 \cdot Q_2 \cdot \mathrm{f}_k \|^2$$

Measure of Smoothness between Blocks

$$+ \lambda_2 \| W_3 \cdot (\mathrm{f}_k - \mathrm{f}_{mc}) \|^2$$

Measure of Temporal Continuity -- Motion Vectors are Known

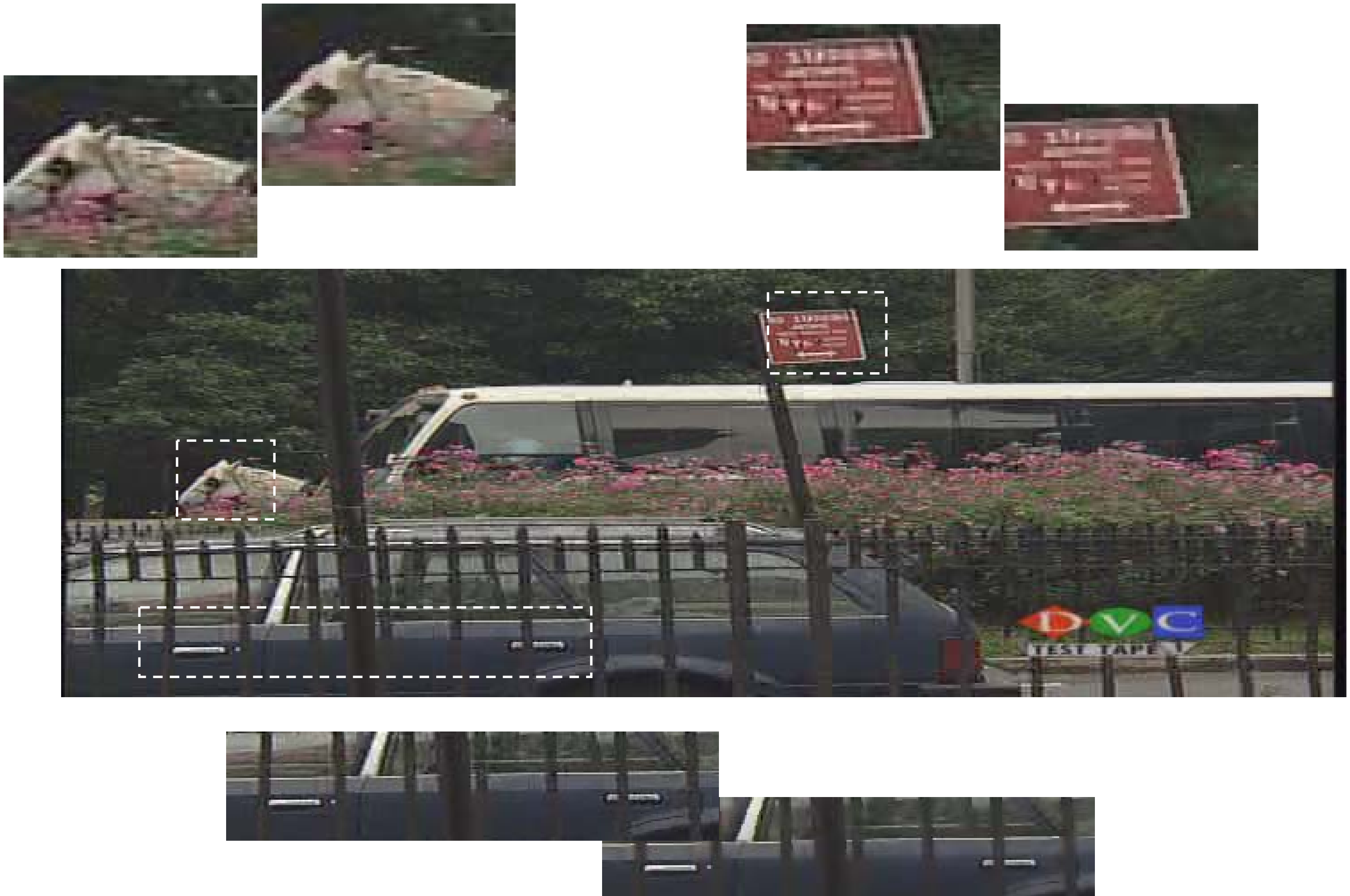
$$s.t.\ \mathrm{DCT}(\mathrm{f}_k) \in S_Q$$

Fidelity to Decoded Image

Experimental Results

P-Frame

Experimental Results



P-Frame

Experimental Results

B-Frame

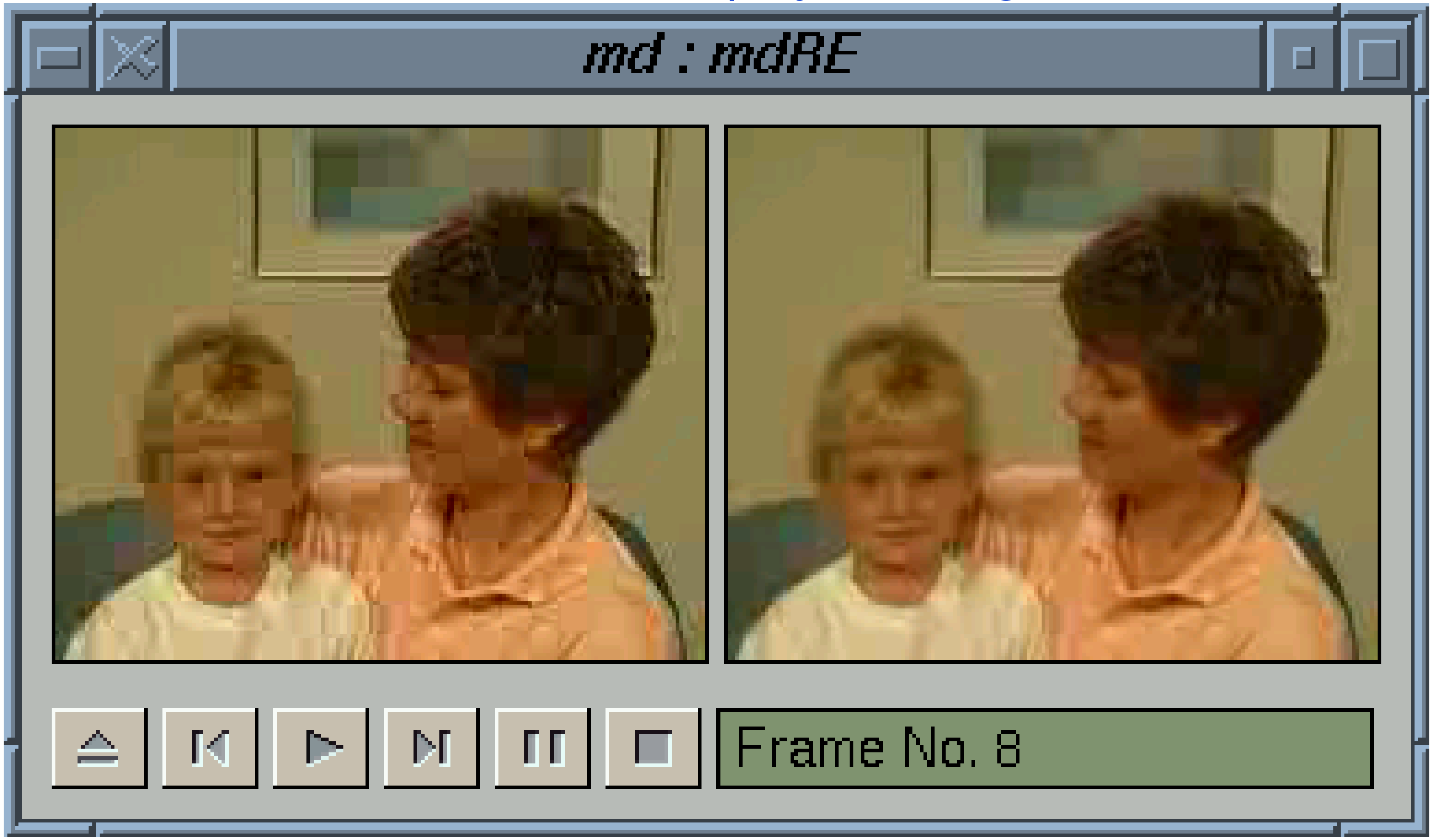
Experimental Results

B-Frame

Experimental Results

Compressed frame by H.261 at 30kbps (10 fps)

Recovered frame by gradient projection algorithm



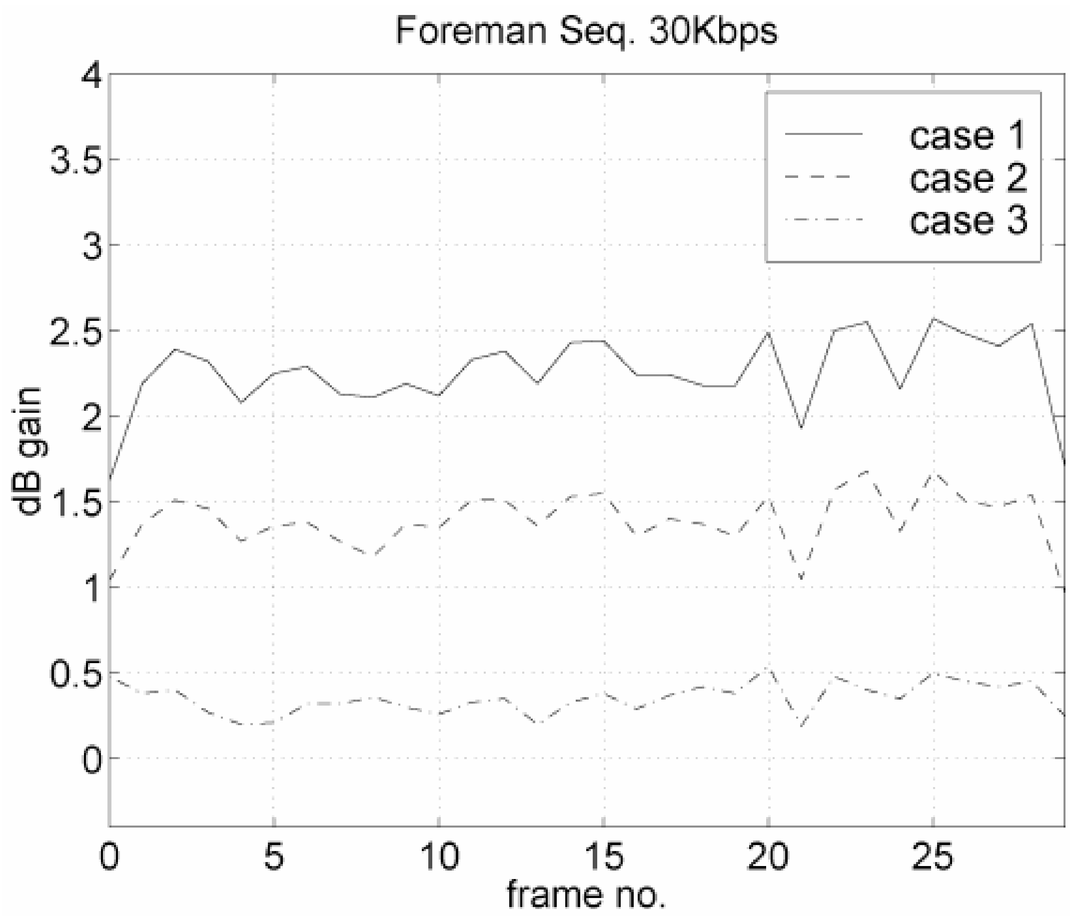
Empirical Performance Bound

Algorithm Performance on *foreman* sequence compressed at 30 kbps:

Case 1: using pixel-basis motion vectors computed from the uncompressed frames

Case 2: using 4×4 pixel basis motion vectors computed from the uncompressed frames

Case 3: using 4×4 pixel basis motion vectors computed from the decompressed frames



Experimental Results

Compressed frame by H.261 at 30kbps

Recovered frame; estimated regularization parameters; motion field estimated from uncompressed frame

Error Concealment Problem Formulation

- Block Transformed Image

$$\mathbf{F} = \mathbf{B}\mathbf{f} \text{ and } \mathbf{f} = \mathbf{B}^{\mathbf{T}}\mathbf{F}$$

- Quantized Version of Transform Coefficients

$$\hat{\mathbf{F}} = \mathbf{Q}[\mathbf{F}]$$

- Representation at Decoder

$$\hat{\mathbf{F}} = \sum_{\mathbf{n}\in\mathbf{R}} \hat{\mathbf{F}}_{\mathbf{n}} + \sum_{\mathbf{n}\in\mathbf{L}} \hat{\mathbf{F}}_{\mathbf{n}}$$

Received coefficients (→ $\mathbf{n}\in\mathbf{R}$) *Lost coefficients* (→ $\mathbf{n}\in\mathbf{L}$)

$$\hat{\mathbf{F}} = (\mathbf{I}_{\mathbf{r}} + \mathbf{I}_{\mathbf{l}})\hat{\mathbf{F}} \Longrightarrow \mathbf{f}_{\mathbf{r}} = \hat{\mathbf{f}} - \mathbf{B}^{\mathbf{T}}\mathbf{I}_{\mathbf{l}}\hat{\mathbf{F}}$$

Problem Formulation (cont'd)

Find $\hat{\mathbf{f}}$ belonging to

$$\| \mathbf{f_r} - \hat{\mathbf{f}} \|^2 \leq \varepsilon^2$$

and

$$\| \mathbf{C} \hat{\mathbf{f}} \|^2 \leq E^2$$

⇨ *Minimize*

$$M(\alpha, \hat{\mathbf{f}}) = \| \mathbf{f_r} - \hat{\mathbf{f}} \|^2 + \alpha \| \mathbf{C} \hat{\mathbf{f}} \|^2$$

$$\text{with } \alpha = \left(\frac{\varepsilon}{E} \right)^2$$

Regularized Iterative Solution

- Transform Domain

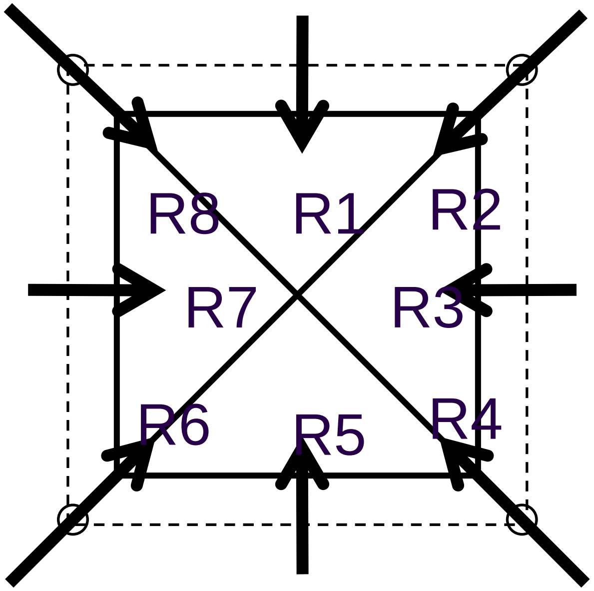
$$\hat{\mathbf{F}}_{\mathbf{k+1}} = \beta\left[K(\alpha)\mathbf{B} - \alpha\mathbf{I_l}\mathbf{B}\mathbf{C}^{\mathbf{T}}\mathbf{C}\right]\mathbf{F_r} + \left[\mathbf{I} - \beta K(\alpha)\right]\hat{\mathbf{F}}_{\mathbf{k}}$$

- Spatial Domain

$$\hat{\mathbf{f}}_{\mathbf{k+1}} = \beta\mathbf{B}^{\mathbf{T}}\left[K(\alpha)\mathbf{B} - \alpha\mathbf{I_l}\mathbf{B}\mathbf{C}^{\mathbf{T}}\mathbf{C}\right]\mathbf{f_r} + \left[\mathbf{I} - \beta\mathbf{B}^{\mathbf{T}}K(\alpha)\mathbf{B}\right]\hat{\mathbf{f}}_{\mathbf{k}}$$

with $K(\alpha) = \mathbf{I} + \alpha\mathbf{I_l}\mathbf{B}\mathbf{C}^{\mathbf{T}}\mathbf{C}\mathbf{B}^{\mathbf{T}}\mathbf{I_l}$

Oriented High Pass Operator



Cell Burst

- All Information in a Block is Lost
  Considerable Discontinuity between
- Burst and Neighboring Blocks
- Replacement of DC Value from
  Neighboring Blocks (α trim mean)

Compressed at .8 bpp

8% error, DC only preserved

Restored 2D Laplacian

Restored oriented smoothness operator

Burst errors

Modified initial image (a=7)

Restored image

Intra Concealment

Described algorithm

Algorithm by Schwab

Intra Concealment

Described algorithm

Algorithm by Schwab

Inter Concealment

average

median

new vector

Discussion

- prior knowledge: critical
- blind restoration: still a formidable problem
- adaptivity (spatial, temporal, frequency, iteration) important
- New applications are driving progress in the field